CEE 271 APPLIED MECHANICS II

Lectures 9 & 10: Equations of Motion
n-t coordinates and cylindrical coordinates

Department of Civil & Environmental Engineering
University of Hawaiʻi at Mānoa

Today's Objectives

- Apply the equation of motion using normal and tangential coordinates.
- Analyze the kinetics of a particle using cylindrical coordinates.

Outline (Pre-Job Brief)

- Equation of Motion using n-t Coordinates
- Equation of Motion using Cylindrical Coordinates
- Examples and Questions
- Summary and Feedback

EOM:
Normal & Tangential Components

Applications

Race track turns are often banked to reduce the frictional forces required to keep the cars from sliding up to the outer rail at high speeds.

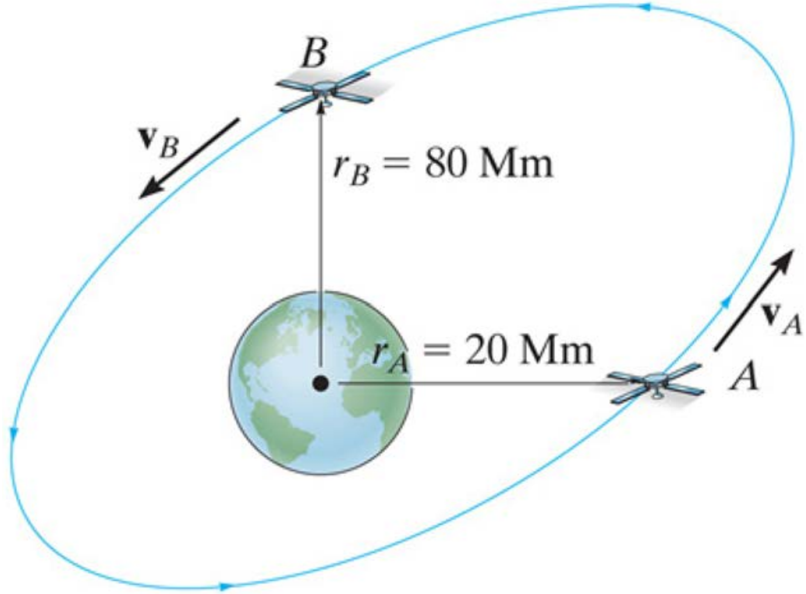
If the car's maximum velocity and a minimum coefficient of friction between the tires and track are specified, how can we determine the minimum banking angle (θ) required to prevent the car from sliding up the track?

Applications (continued)

This picture shows a ride at the amusement park. The hydraulically-powered arms turn at a constant rate, which creates a centrifugal force on the riders.

We need to determine the smallest angular velocity of cars A and B such that the passengers do not lose contact with their seat. What parameters are needed for this calculation?

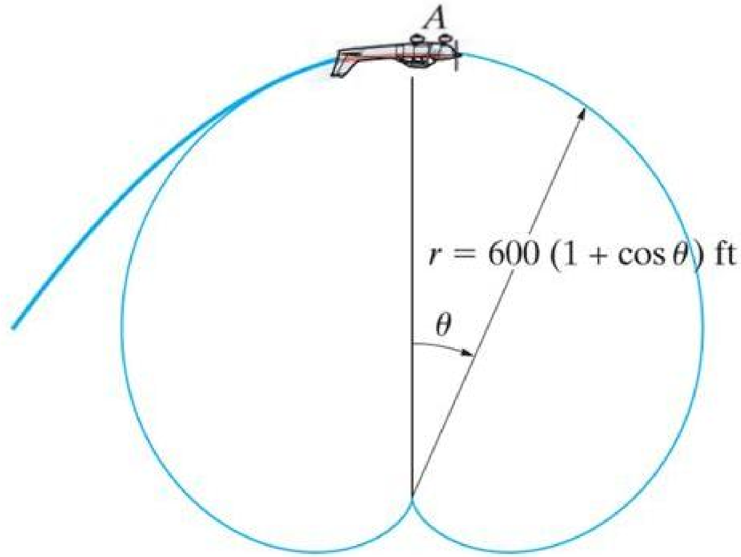
Applications (continued)



Satellites are held in orbit around the earth by using the earth's gravitational pull as the centripetal force – the force acting to change the direction of the satellite's velocity.

Knowing the radius of orbit of the satellite, we need to determine the required speed of the satellite to maintain this orbit. What equation governs this situation?

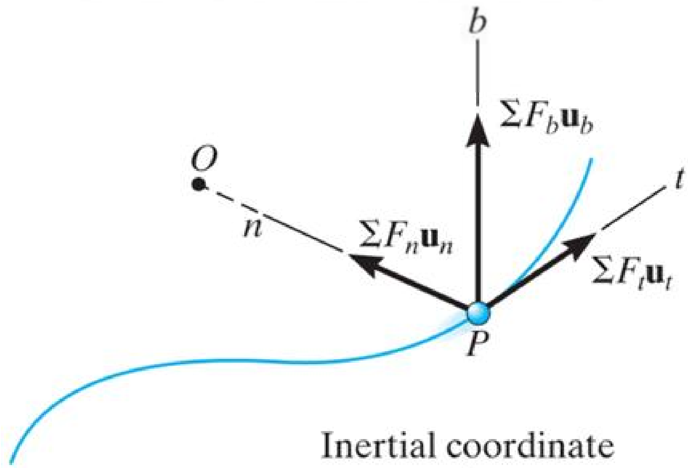
Applications (continued)



When an airplane executes the vertical loop shown above, the centrifugal force causes the normal force (apparent weight) on the pilot to be smaller than her actual weight.

How would you calculate the velocity necessary for the pilot to experience weightlessness at A?

Normal & Tangential Coordinates



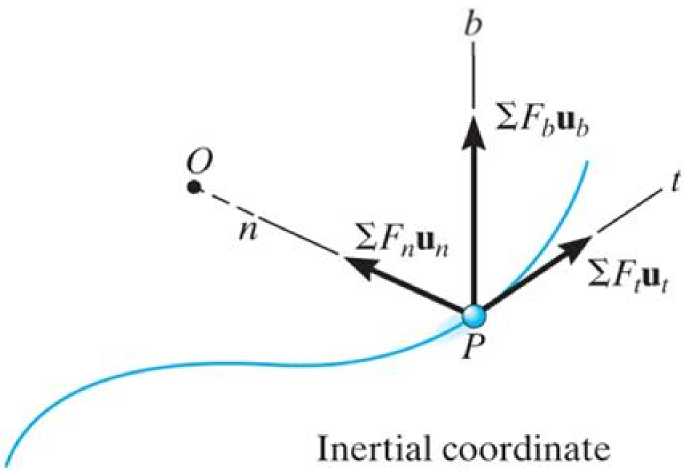
Inertial coordinate system

When a particle moves along a curved path, it may be more convenient to write the equation of motion in terms of normal and tangential coordinates.

The normal direction (n) *always* points toward the path's center of curvature. In a circle, the center of curvature is the center of the circle.

The tangential direction (t) is tangent to the path, usually set as positive in the direction of motion of the particle.

Equation of Motion



Inertial coordinate system

Since the equation of motion is a vector equation , $\sum \boldsymbol{F} = m\mathbf{a}$, it may be written in terms of the n & t coordinates as

$$\sum F_t \boldsymbol{u}_t + \sum F_n \boldsymbol{u}_n + \sum F_b \boldsymbol{u}_b = m\mathbf{a}_t + m\mathbf{a}_n$$

Here $\sum F_t$ & $\sum F_n$ are the sums of the force components acting in the t & n directions, respectively.

This vector equation will be satisfied provided the individual components on each side of the equation are equal, resulting in the two scalar equations: $\sum F_t = ma_t$ and $\sum F_n = ma_n$.

Since there is no motion in the binormal (b) direction, we can also write $\sum F_b = 0$.

Normal & Tangential Accelerations

The tangential acceleration, $a_t = dv/dt$, represents the time rate of change in the magnitude of the velocity. Depending on the direction of $\sum F_t$, the particle's speed will either be increasing or decreasing.

The normal acceleration, $a_n = v^2/\rho$, represents the time rate of change in the direction of the velocity vector. Remember, a_n always acts toward the path's center of curvature. Thus, $\sum F_n$ will always be directed toward the center of the path.

Recall, if the path of motion is defined as $y = f(x)$, the radius of curvature at any point can be obtained from

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Solving Problems

Use n-t coordinates when a particle is moving along a known, curved path.

Establish the n-t coordinate system on the particle.

Draw free-body and kinetic diagrams of the particle. The normal acceleration (a_n) always acts “inward” (the positive n-direction). The tangential acceleration (a_t) may act in either the positive or negative t direction.

Apply the equations of motion in scalar form and solve.

It may be necessary to employ the kinematic relations:

$$a_t = dv/dt = v\ dv/ds \qquad a_n = v^2/\rho$$

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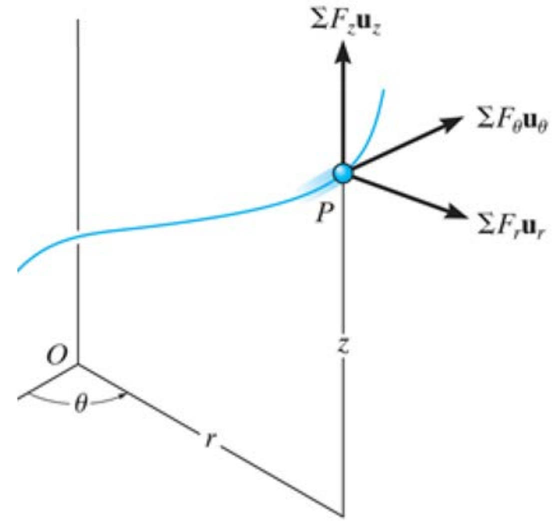
EOM: Cylindrical Components

Cylindrical Coordinates

This approach to solving problems has some external similarity to the normal & tangential method just studied. However, the path may be more complex or the problem may have other attributes that make it desirable to use cylindrical coordinates.



Equilibrium equations or "Equations of Motion" in cylindrical coordinates (using r, θ , and z coordinates) may be expressed in scalar form as:

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = m\,a_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

$$\sum F_z = ma_z = m\ddot{z}$$

Cylindrical Coordinates

If the particle is constrained to move only in the $r - \theta$ plane (i.e., the z coordinate is constant), then only the first two equations are used (as shown below). The coordinate system in such a case becomes a polar coordinate system. In this case, the path is only a function of θ.

$$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$
$$\sum F_\theta = m\,a_\theta = m(r\ddot{\theta} - 2\dot{r}\dot{\theta})$$

Note that a fixed coordinate system is used, not a "body-centered" system as used in the n – t approach.

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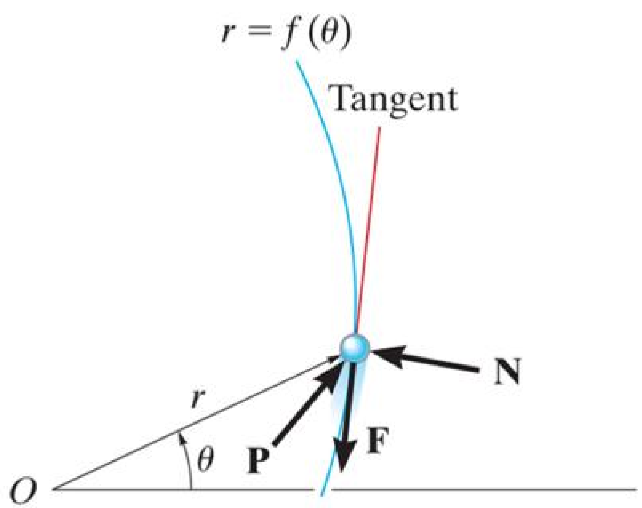
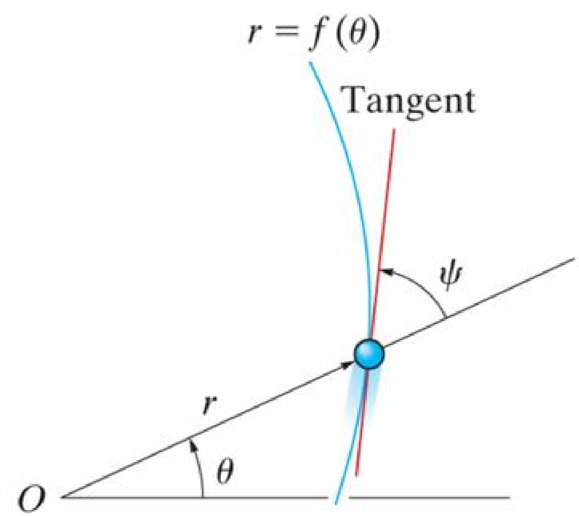
Tangential and Normal Forces

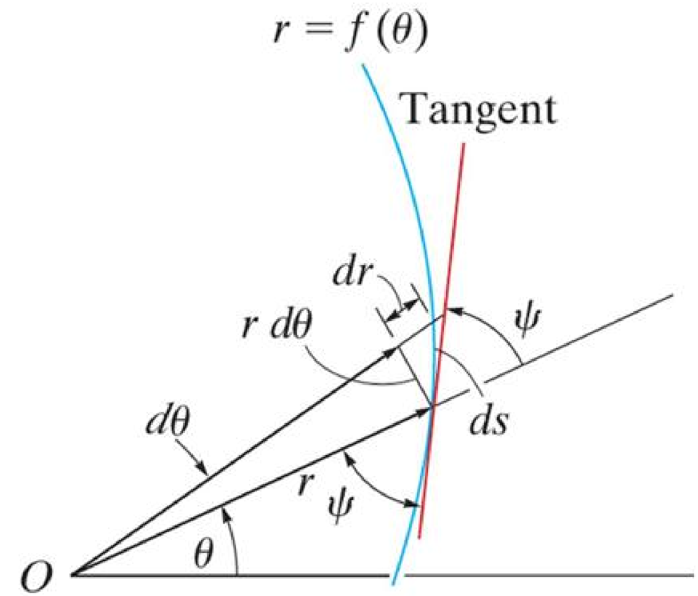
If a force ***P*** causes the particle to move along a path defined by r = f (θ), the normal force ***N*** exerted by the path on the particle is always perpendicular to the path's tangent. The frictional force ***F*** always acts along the tangent in the opposite direction of motion. The directions of ***N*** and ***F*** can be specified relative to the radial coordinate by using angle ψ.





Determination of Angle ψ

The angle ψ, defined as the angle between the extended radial line and the tangent to the curve, can be required to solve some problems. It can be determined from the following relationship.



$$\tan\psi = \frac{r d\theta}{dr} = \frac{r}{dr/d\theta}$$

If ψ is positive, it is measured counterclockwise from the radial line to the tangent. If it is negative, it is measured clockwise.

Examples & Questions

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