CEE 271 APPLIED MECHANICS II

Lectures 7 & 8: Equations of Motion

Department of Civil & Environmental Engineering
University of Hawaiʻi at Mānoa

Today's Objectives

- Write the equation of motion for an accelerating body.
- Draw the free-body and kinetic diagrams for an accelerating body.
- Apply Newton's second law to determine forces and accelerations for particles in rectilinear motion.

Outline (Pre-Job Brief)

- Newton's Laws of Motion
- Newton's Law of Gravitational Attraction
- Equation of Motion for a Particle or System of Particles
- Equation of Motion using Rectangular Coordinates
- Examples and Questions
- Summary and Feedback

Equation of Motion

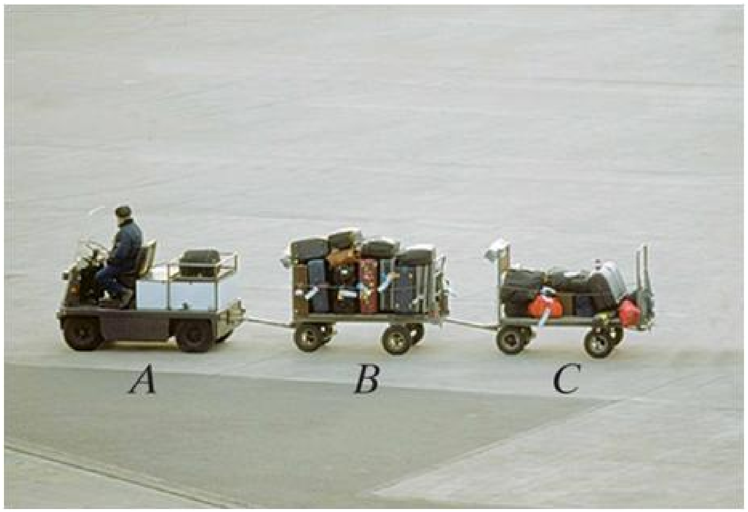
Applications

The motion of an object depends on the forces acting on it.

A parachutist relies on the atmospheric drag resistance force of her parachute to limit her velocity.

Knowing the drag force, how can we determine the acceleration or velocity of the parachutist at any point in time? This has some importance when landing!

Applications (continued)



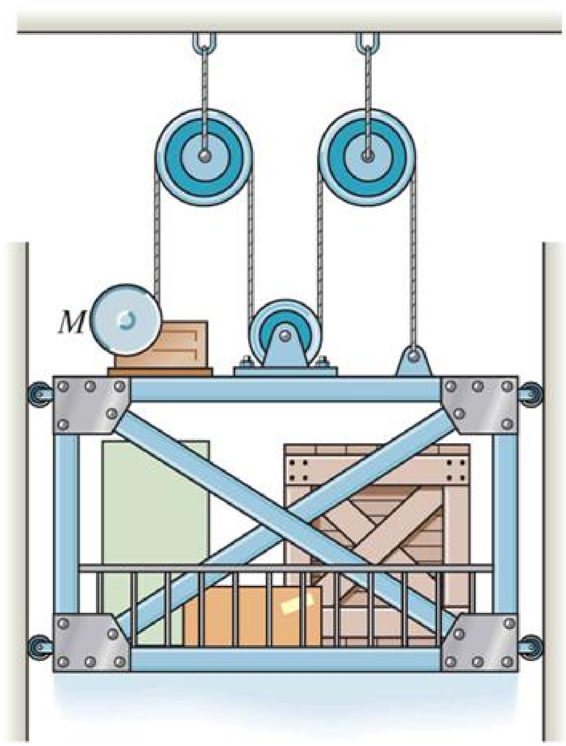
The baggage truck A tows a cart B, and a cart C.

If we know the frictional force developed at the driving wheels of the truck, could we determine the acceleration of the truck?

How?

Can we also determine the horizontal force acting on the coupling between the truck and cart *B*? This is needed when designing the coupling (or understanding why it failed).

Applications (continued)

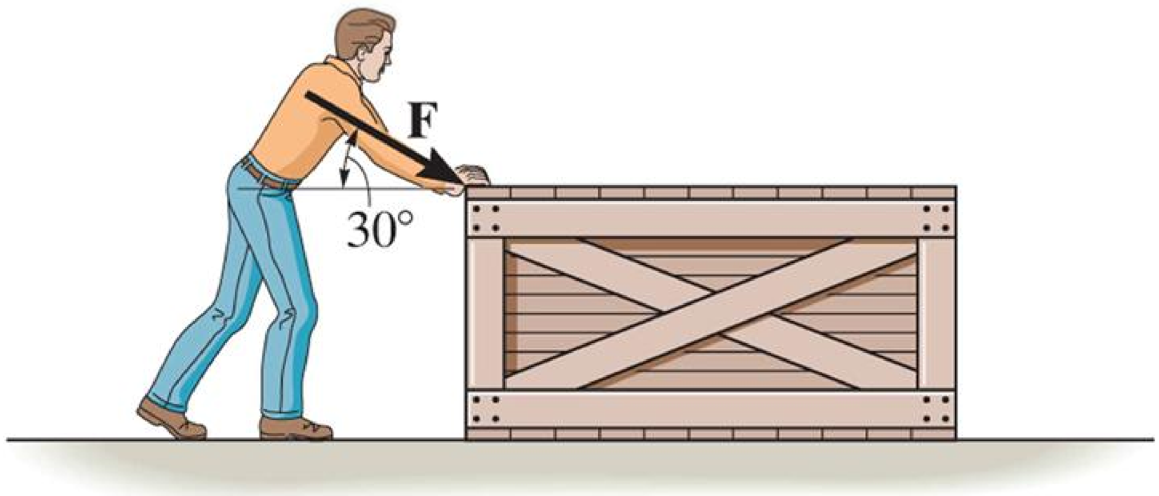


A freight elevator is lifted using a motor attached to a cable and pulley system as shown.

How can we determine the tension force in the cable required to lift the elevator and load at a given acceleration? This is needed to decide what size cable should be used.

Is the tension force in the cable greater than the weight of the elevator and its load?

Applications (continued)

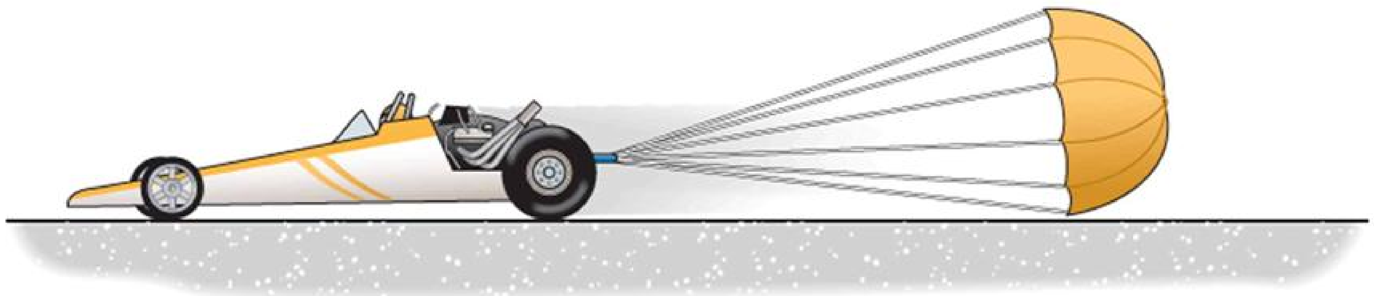


If a man is trying to move a 100 lb crate, how large a force F must he exert to start moving the crate? What factors influence how large this force must be to start moving the crate?

If the crate starts moving, is there acceleration present?

What would you have to know before you could find these answers?

Applications (continued)



Objects that move in air (or other fluid) have a drag force acting on them. This drag force is a function of velocity.

If the dragster is traveling with a known velocity and the magnitude of the opposing drag force at any instant is given as a function of velocity, can we determine the time and distance required for dragster to come to a stop if its engine is shut off? How ?

Newton's Laws of Motion

The motion of a particle is governed by Newton's three laws of motion.

First Law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

Second Law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

Third Law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

Laws of Motion (continued)

The first and third laws were used in developing the concepts of statics. Newton's second law forms the basis of the study of dynamics.

Mathematically, Newton's second law of motion can be written

$$\boldsymbol{F} = m\mathbf{a}$$

where $\boldsymbol{F}$ is the resultant unbalanced force acting on the particle, and $\mathbf{a}$ is the acceleration of the particle. The positive scalar m is called the mass of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).

Gravitational Attraction

Any two particles or bodies have a mutually attractive gravitational force acting between them. Newton postulated the law governing this gravitational force as

$$F = G(m_1 m_2/r^2)$$

where F = force of attraction between the two bodies,
G = universal constant of gravitation ,
m_1, m_2 = mass of each body, and
r = distance between centers of the two bodies.

When near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the body. This force is called the weight of the body.

Mass and Weight

It is important to understand the difference between the mass and weight of a body!

Mass is an absolute property of a body. It is independent of the gravitational field in which it is measured. The mass provides a measure of the resistance of a body to a change in velocity, as defined by Newton's second law of motion ($m = \boldsymbol{F}/\mathbf{a}$).

The weight of a body is not absolute, since it depends on the gravitational field in which it is measured. Weight is defined as

$$W = mg$$

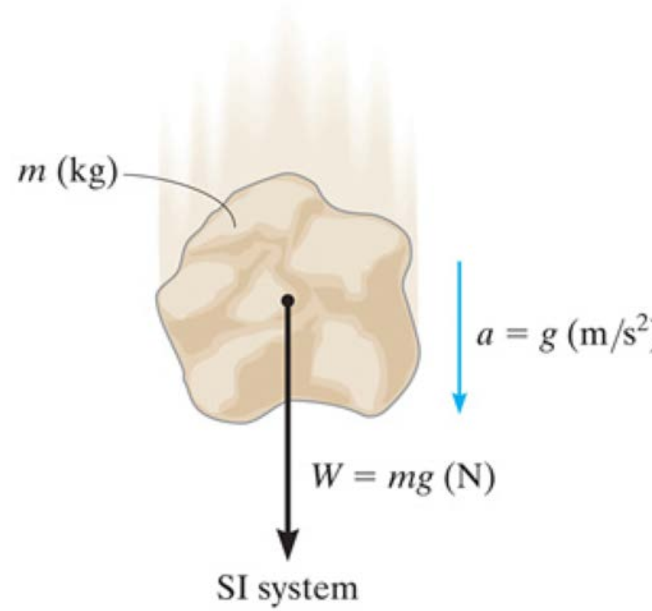
where g is the acceleration due to gravity.

Units: SI vs. FPS

SI system: In the SI system of units, mass is a base unit and weight is a derived unit.

Typically, mass is specified in kilograms (kg), and weight is calculated from W = mg.

If the gravitational acceleration (g) is specified in units of m/s^2, then the weight is expressed in Newtons (N).



On the earth's surface, g can be taken as $g = 9.81\ m/s^2$.

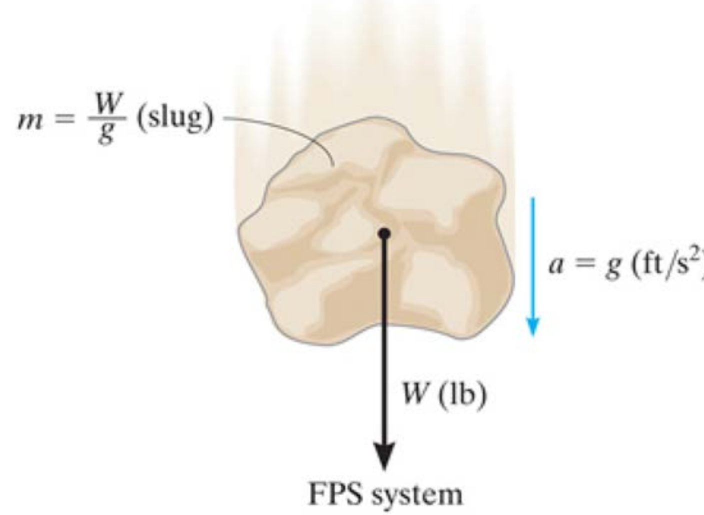
$W\ (N) = m\ (kg)\ g\ (m/s^2) \Rightarrow N = kg \cdot m/s^2$

Units: SI vs. FPS (continued)

FPS System: In the FPS system of units, weight is a base unit and mass is a derived unit.

Weight is typically specified in pounds (lb), and mass is calculated from m = W/g.

If g is specified in units of ft/s^2, then the mass is expressed in slugs.



On the earth's surface, g is approximately 32.2 ft/s^2.

m (slugs) = W (lb)/g (ft/s^2) => slug = $lb \cdot s^2/ft$

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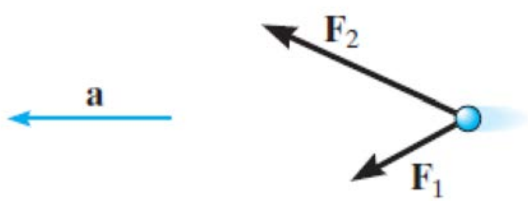
Equation of Motion

The motion of a particle is governed by Newton's second law, relating the unbalanced forces on a particle to its acceleration. If more than one force acts on the particle, the equation of motion can be written

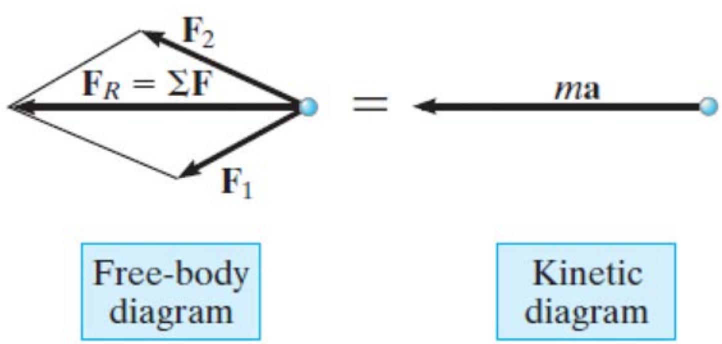
$$\sum \boldsymbol{F} = \boldsymbol{F}_R = m\mathbf{a}$$

where $\boldsymbol{F}_R$ is the resultant force, which is a vector summation of all the forces.

To illustrate the equation, consider a particle acted on by two forces.



First, draw the particle's free-body diagram, showing all forces acting on the particle. Next, draw the kinetic diagram, showing the inertial force m**a** acting in the same direction as the resultant force $\boldsymbol{F}_R$.



Inertial Frame of Reference

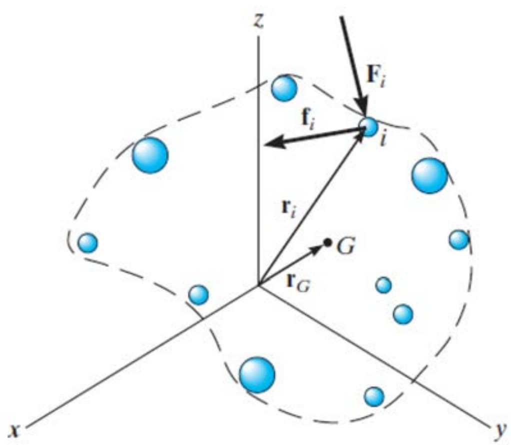
This equation of motion is only valid if the acceleration is measured in a Newtonian or inertial frame of reference. What does this mean?

For problems concerned with motions at or near the earth's surface, we typically assume our "inertial frame" to be fixed to the earth. We neglect any acceleration effects from the earth's rotation.

For problems involving satellites or rockets, the inertial frame of reference is often fixed to the stars.

EOM System of Particles

The equation of motion can be extended to include systems of particles. This includes the motion of solids, liquids, or gas systems.



Inertial coordinate system

As in statics, there are internal forces and external forces acting on the system. What is the difference between them?

Using the definitions of $m = \sum m_i$ as the total mass of all particles and $\mathbf{a}_G$ as the acceleration of the center of mass G of the particles, then $m\,\mathbf{a}_G = \sum m_i\,\mathbf{a}_i$.

The text shows the details, but for a system of particles: $\sum \boldsymbol{F} = m\,\mathbf{a}_G$ where $\sum \boldsymbol{F}$ is the sum of the external forces acting on the entire system.

Key Points

1) Newton's second law is a "law of nature"-- experimentally proven, not the result of an analytical proof.

2) Mass (property of an object) is a measure of the resistance to a change in velocity of the object.

3) Weight (a force) depends on the local gravitational field. Calculating the weight of an object is an application of $F = m\,a$, i.e., $W = m\,g$.

4) Unbalanced forces cause the acceleration of objects. This condition is fundamental to all dynamics problems!

Procedure for EOM

1) Select a convenient inertial coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.

2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components.

3) Draw the kinetic diagram, showing the particle's inertial force, m**a**. Resolve this vector into its appropriate components.

4) Apply the equations of motion in their scalar component form and solve these equations for the unknowns.

5) It may be necessary to apply the proper kinematic relations to generate additional equations.

Rectangular Coordinates

The equation of motion, $\boldsymbol{F} = m\,\boldsymbol{a}$, is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities, or mass. Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

$$\sum \boldsymbol{F} = m\boldsymbol{a} \quad \text{or} \quad \sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j} + \sum F_z \boldsymbol{k} = m(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k})$$

or, as scalar equations, $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$.

Procedure for Analysis

- **Free Body Diagram (always critical!!)**

  Establish your coordinate system and draw the particle's free body diagram showing only external forces. These external forces usually include the weight, normal forces, friction forces, and applied forces.

  Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic linear spring, a spring force equal to '*k s*' should be included on the FBD.

- **Kinetics Diagram (always critical!!)**

  Show the 'm**a**' vector (sometimes called the inertial force) on a separate diagram.

Procedure for Analysis

- **Equations of Motion**

If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the scalar form of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a vector analysis is often best.

A Cartesian vector formulation of the second law is

$$\sum \boldsymbol{F} = m\mathbf{a} \quad \text{or}$$

$$\sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j} + \sum F_z \boldsymbol{k} = m(a_x \boldsymbol{i} + a_y \boldsymbol{j} + a_z \boldsymbol{k})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

Procedure for Analysis

- **Kinematics**

The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.

Any of the kinematics tools learned in Chapter 12 may be needed to solve a problem.

Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!

Examples & Questions

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