CEE 271 APPLIED MECHANICS II

Lecture 6: Dependent Motion

Department of Civil & Environmental Engineering
University of Hawaiʻi at Mānoa

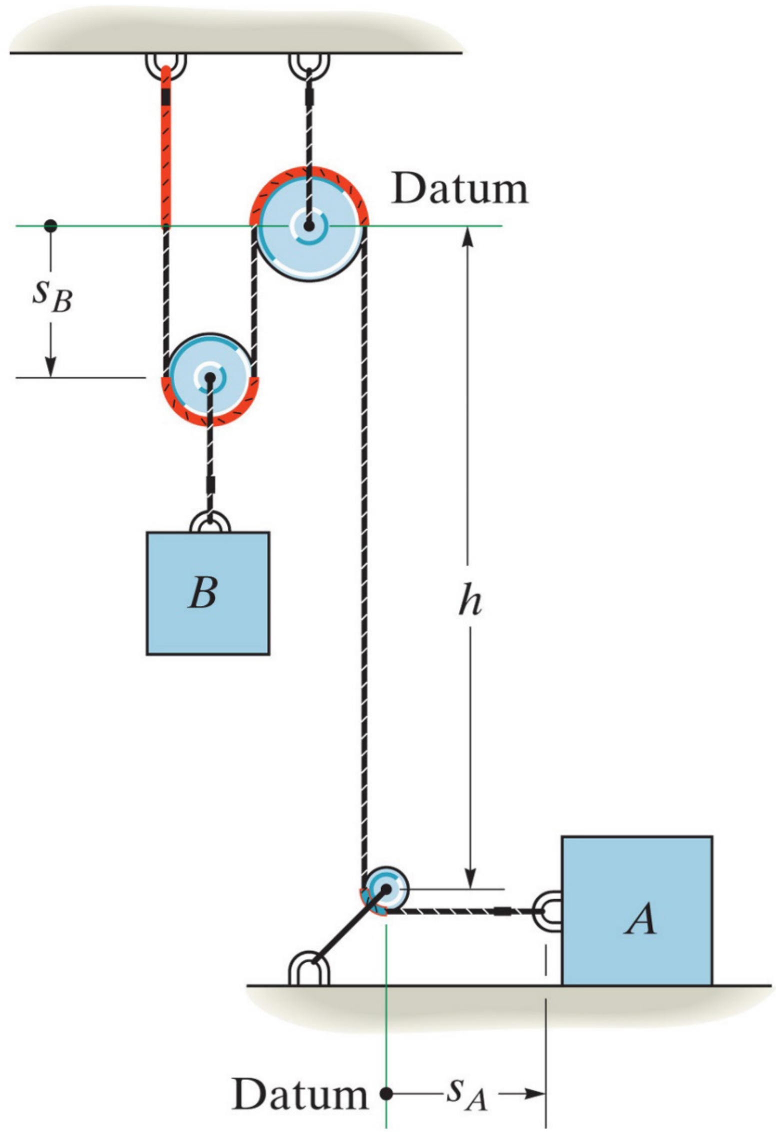
Today's Objectives

- Relate the positions, velocities, and accelerations of particles undergoing dependent motion.

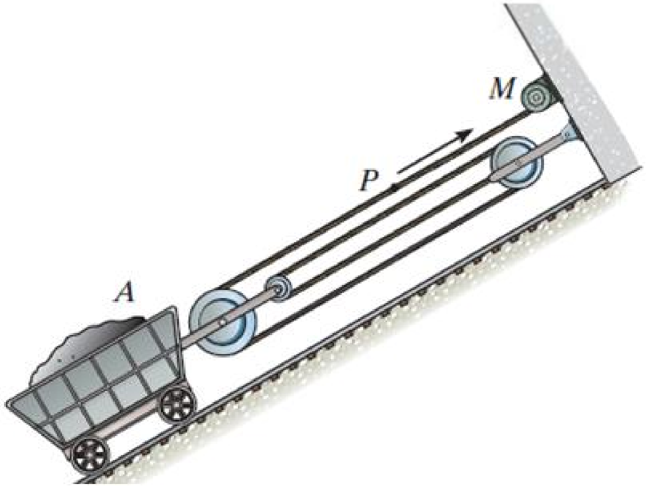
Outline (Pre-Job Brief)

- Dependent Motion
- Relative Motion
- Examples and Questions
- Summary and Feedback

Dependent Motion



Applications

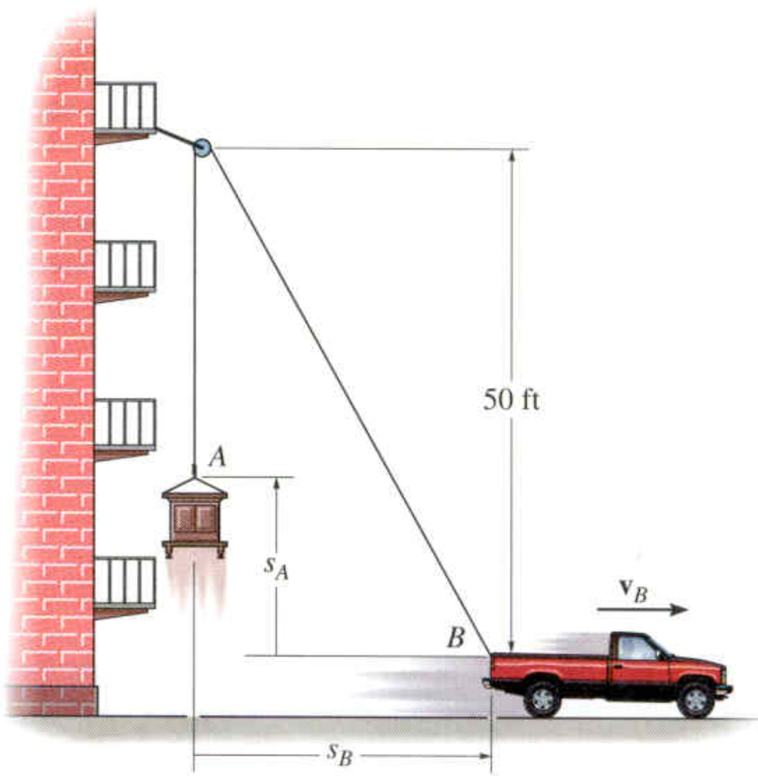


The cable and pulley system shown can be used to modify the speed of the mine car, A, relative to the speed of the motor, M.

It is important to establish the relationships between the various motions in order to determine the power requirements for the motor and the tension in the cable.

For instance, if the speed of the cable (P) is known because we know the motor characteristics, how can we determine the speed of the mine car? Will the slope of the track have any impact on the answer?

Applications (continued)

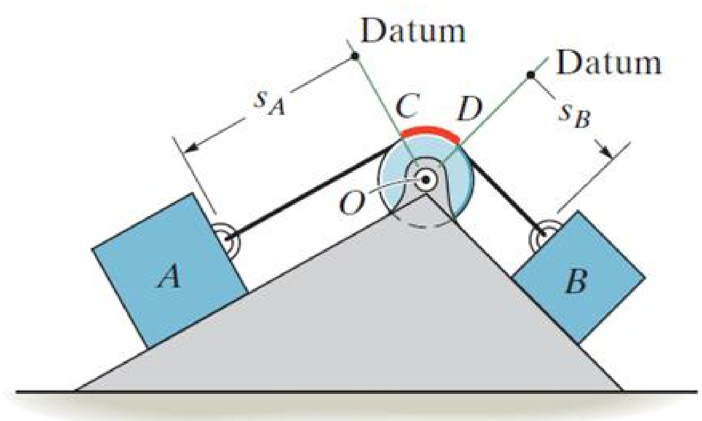


Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on both the weight and the acceleration of the cabinet.

How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?

Dependent Motion

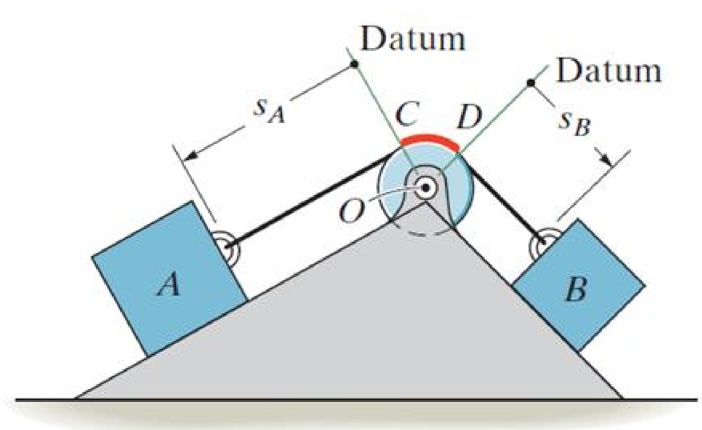
In many kinematics problems, the motion of one object will depend on the motion of another object.



The blocks in this figure are connected by an inextensible cord wrapped around a pulley.
If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining position coordinates, s_A and s_B. Each coordinate axis is defined from a fixed point or datum line, measured positive along each plane in the direction of motion of each block.

Dependent Motion (continued)



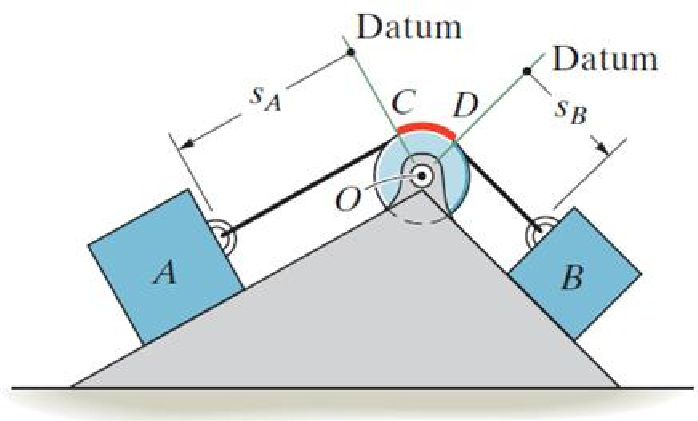
In this example, position coordinates s_A and s_B can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the cord has a fixed length, the position coordinates s_A and s_B are related mathematically by the equation

$$s_A + l_{CD} + s_B = l_T$$

Here l_T is the total cord length and l_{CD} is the length of cord passing over the arc CD on the pulley.

Dependent Motion (continued)



The velocities of blocks A and B can be related by differentiating the position equation. Note that l_{CD} and l_T remain constant, so $dl_{CD}/dt = dl_T/dt = 0$

$$ds_A/dt + ds_B/dt = 0 \quad \Rightarrow \quad v_B = -v_A$$

The negative sign indicates that as A moves down the incline (positive s_A direction), B moves up the incline (negative s_B direction).

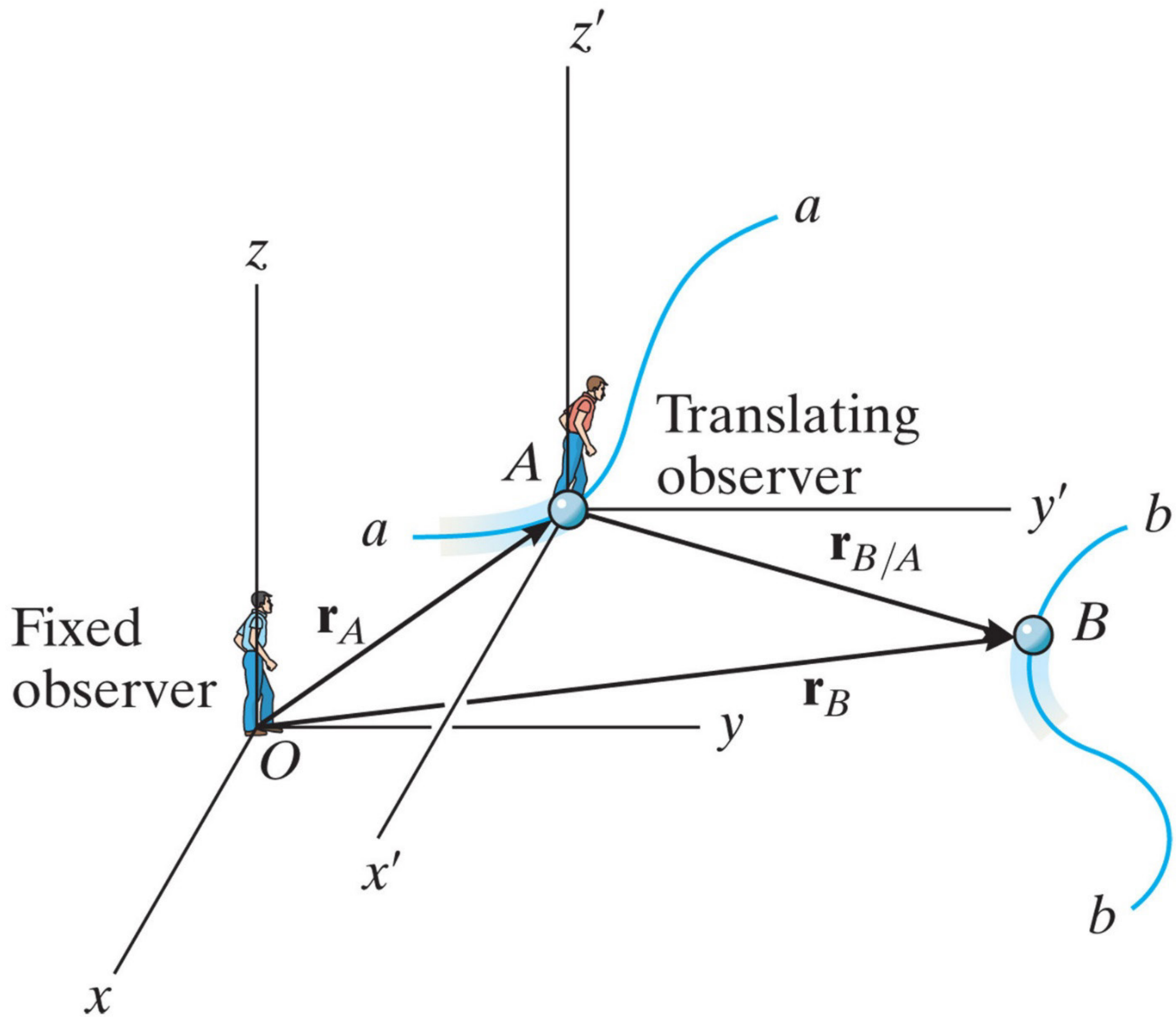
Accelerations can be found by differentiating the velocity expression. Prove to yourself that $a_B = -a_A$.

Dependent Motion: Procedures

These procedures can be used to relate the dependent motion of particles moving along rectilinear paths (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define position coordinates from fixed datum lines, along the path of each particle. Different datum lines can be used for each particle.
2. Relate the position coordinates to the cord length. Segments of cord that do not change in length during the motion may be left out.
3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. Separate equations are written for each cord.
4. Differentiate the position coordinate equation(s) to relate velocities and accelerations. Keep track of signs!

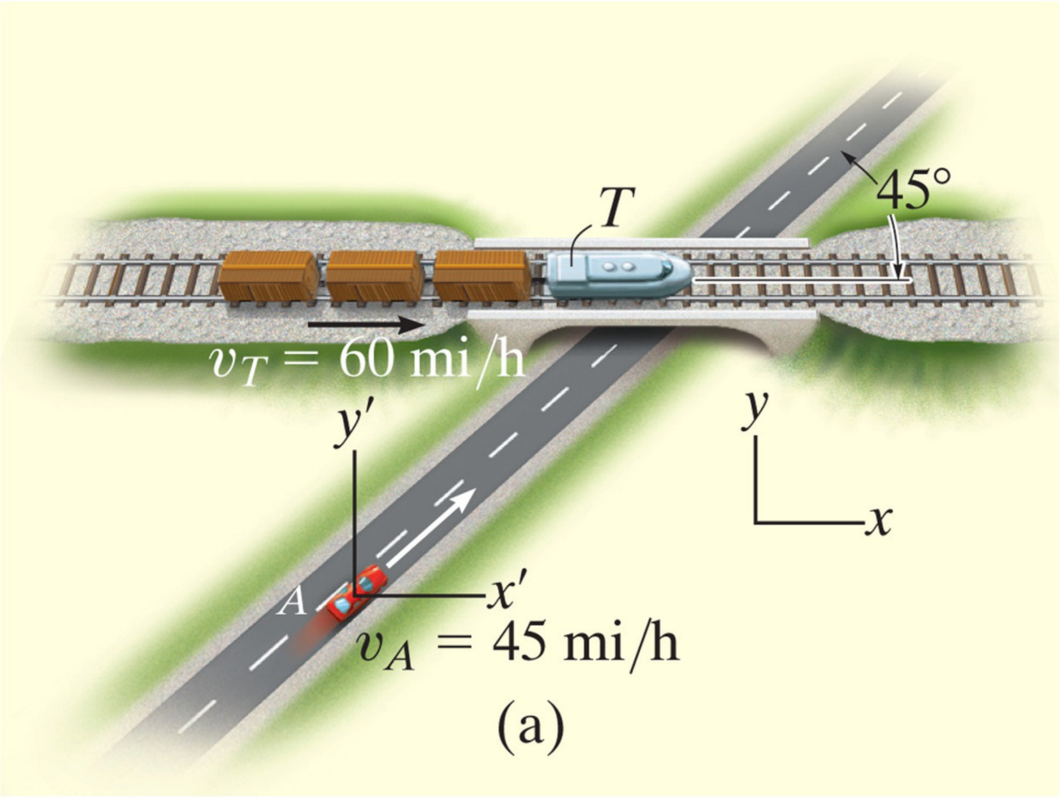
Relative Motion



Relative Motion

Sometimes we need to know the motion of one object relative to another moving object.

What are the velocity and acceleration of the train relative to the car, or vice versa?



(a)

Relative Motion

In the figure, the absolute motions of particles A and B are measured from a common fixed origin, O.

The motion of B relative to the translating observer at A can now be determined by vector addition.

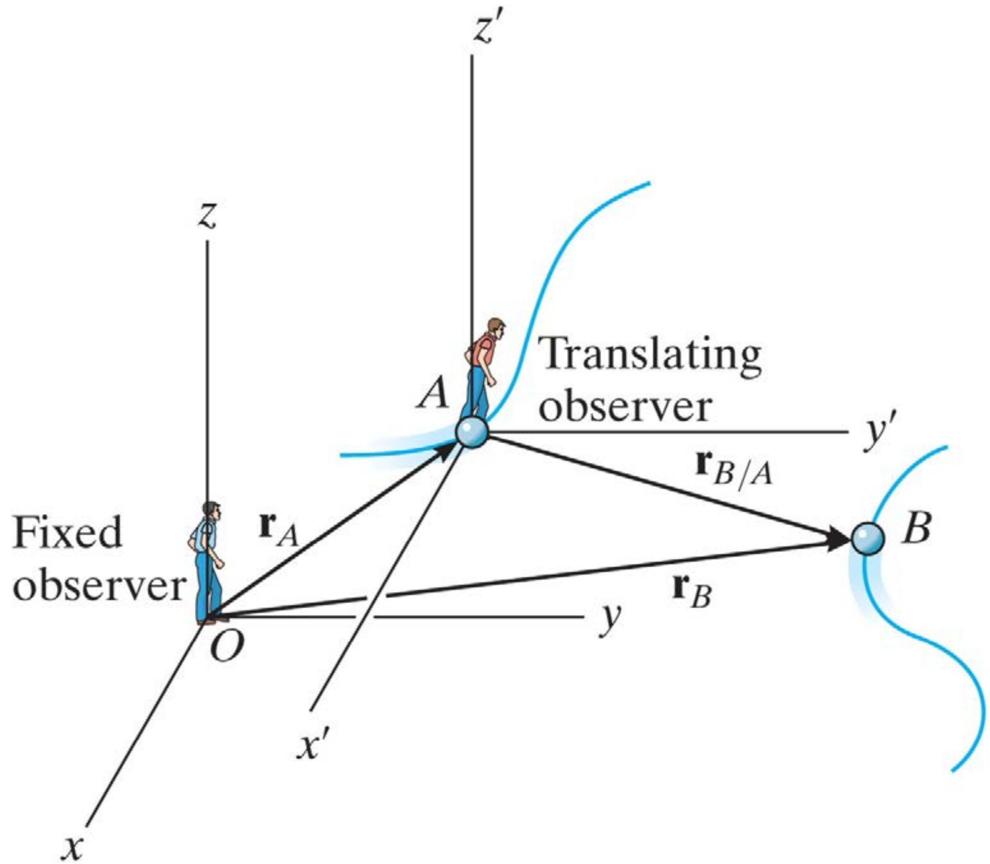


Figure: 12_042

Relative Motion

It is important that the coordinate system of the the translating observer does not rotate relative to the stationary coordinate system.

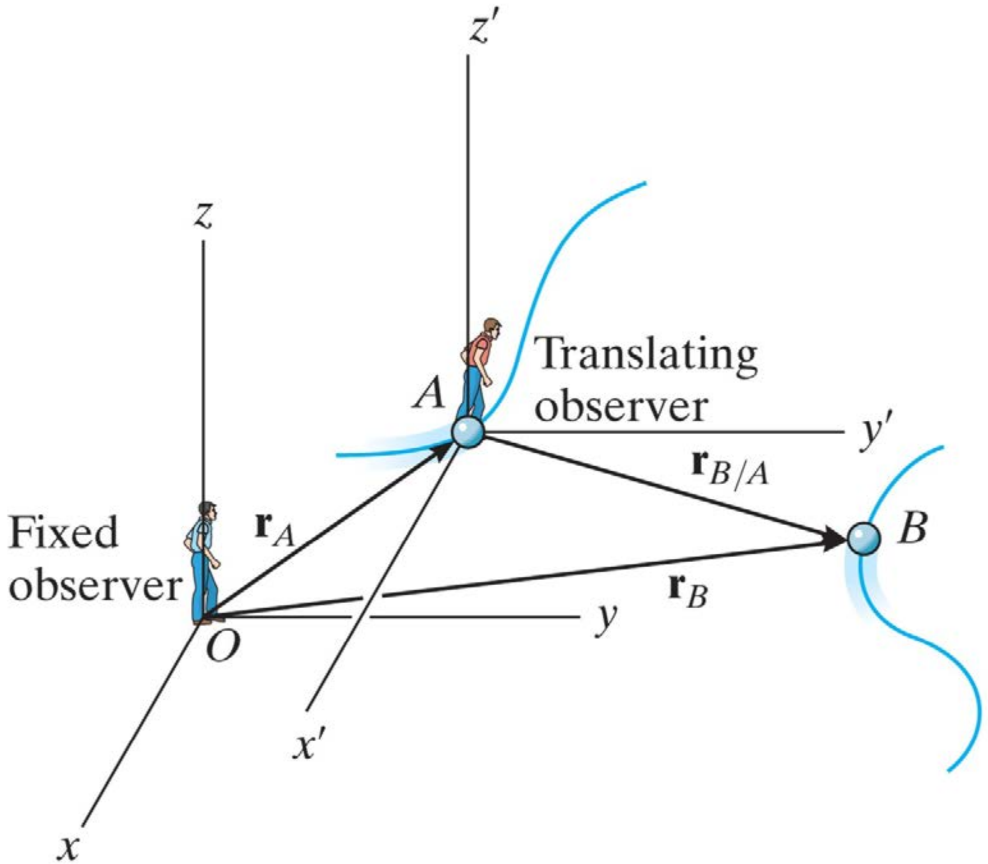


Figure: 12_042

Relative Motion

Vector Addition:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Therefore, the motion of B relative to the translating observer at A is:

$$\mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$$

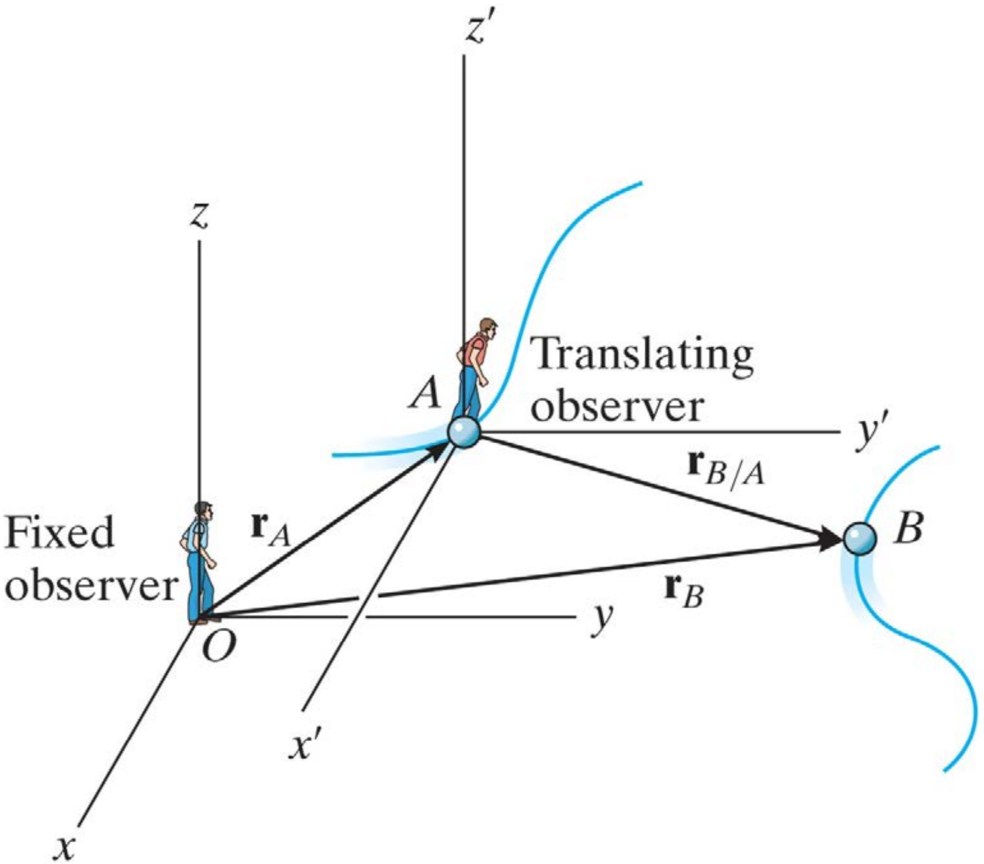


Figure: 12_042

Relative Motion

Vector Addition:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Therefore, taking time derivatives gives:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

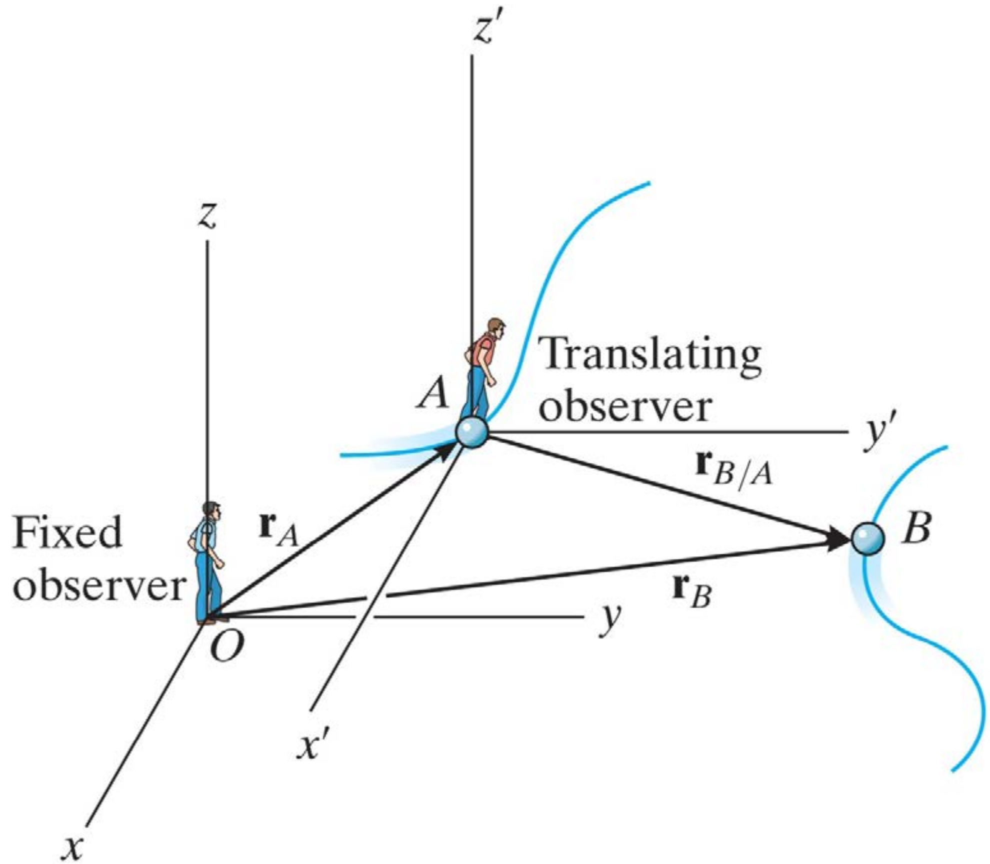


Figure: 12_042

Examples & Questions

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