



UNIVERSITY  
*of* HAWAI'I®  
MĀNOA

# CEE 271 APPLIED MECHANICS II

## Lecture 2: Rectilinear Erratic Motion

Department of Civil & Environmental Engineering  
University of Hawai'i at Mānoa



# Today's Objectives

- Determine position, velocity, and acceleration of a particle using graphs

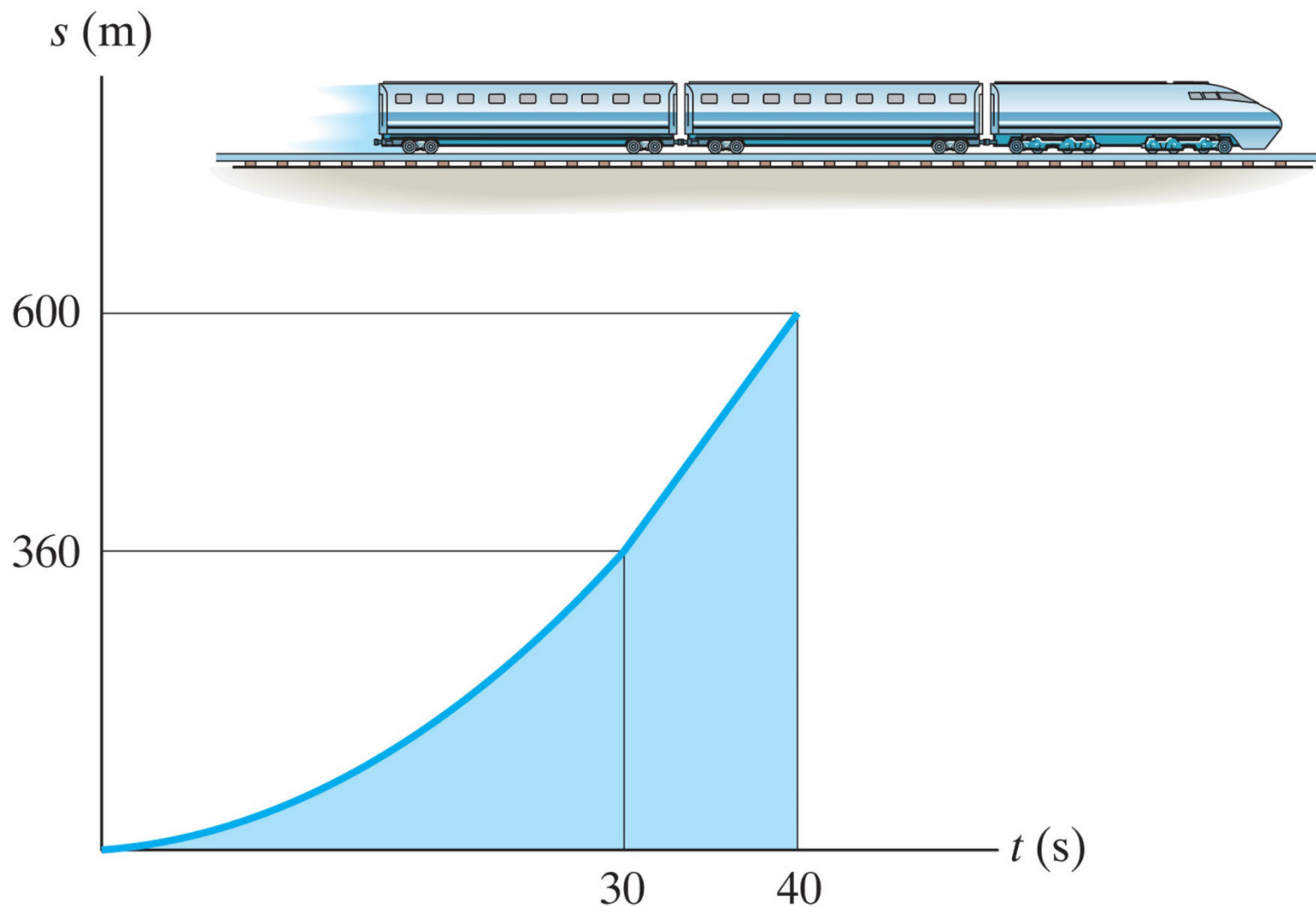


# Outline (Pre-Job Brief)

- Erratic Motion
- Graphs for  $s$ - $t$ ,  $v$ - $t$ ,  $a$ - $t$ ,  $a$ - $s$ ,  $v$ - $s$
- Examples and Questions
- Summary and Feedback

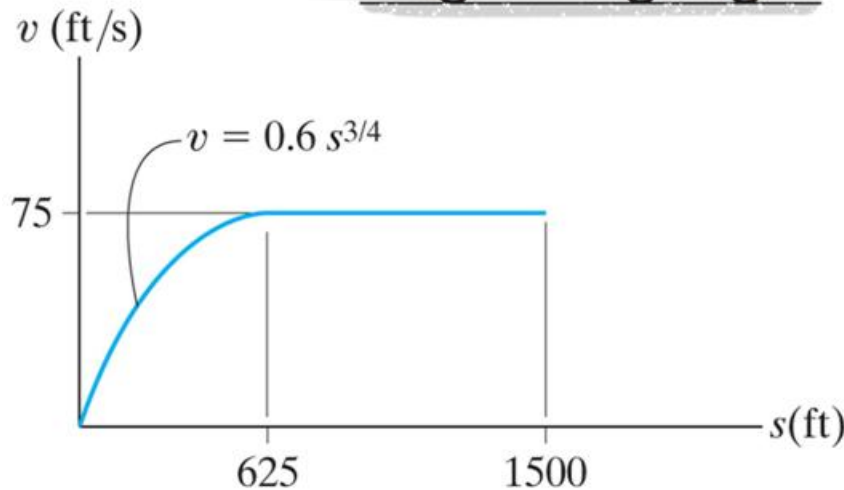
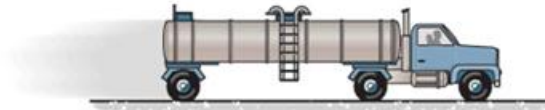


# Rectilinear Erratic Motion





# Applications

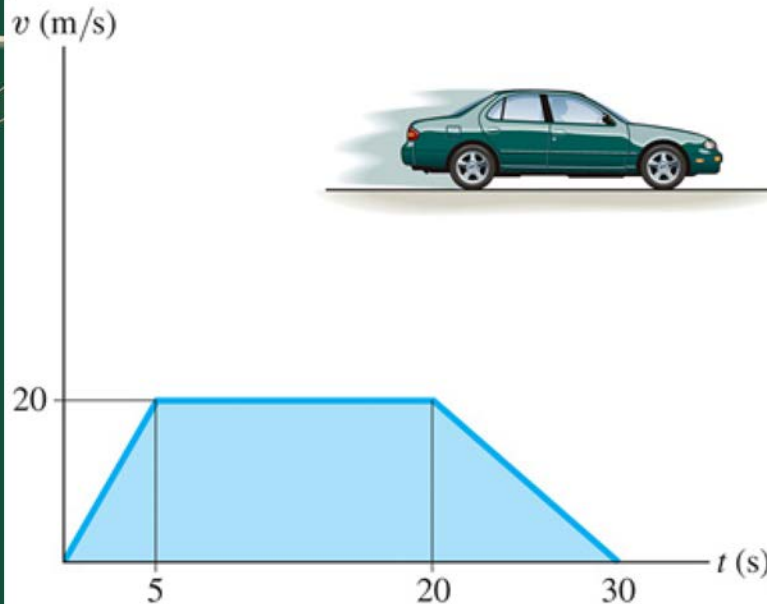


In many experiments, a velocity versus position ( $v$ - $s$ ) profile is obtained.

If we have a  $v$ - $s$  graph for the tank truck, how can we determine its acceleration at position  $s = 1500$  feet?



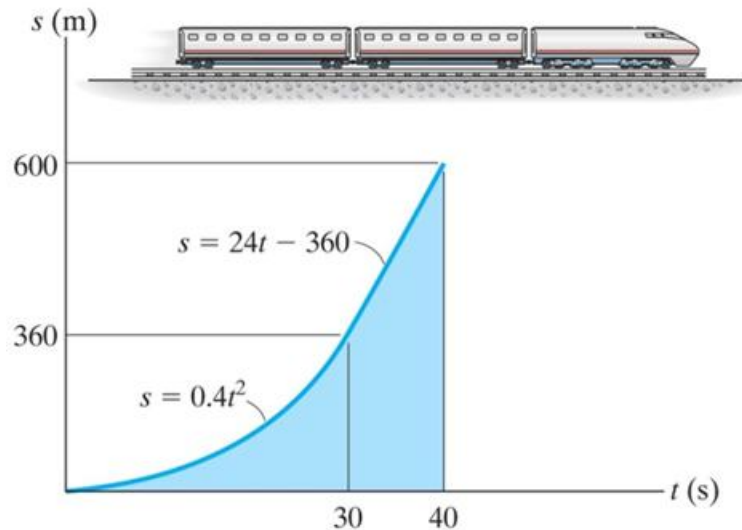
# Applications (continued)



The velocity of a car is recorded from a experiment. The car starts from rest and travels along a straight track.

If we know the v-t plot, how can we determine the distance the car traveled during the time interval  $0 < t < 30$  s or  $15 < t < 25$  s?

# Erratic Motion



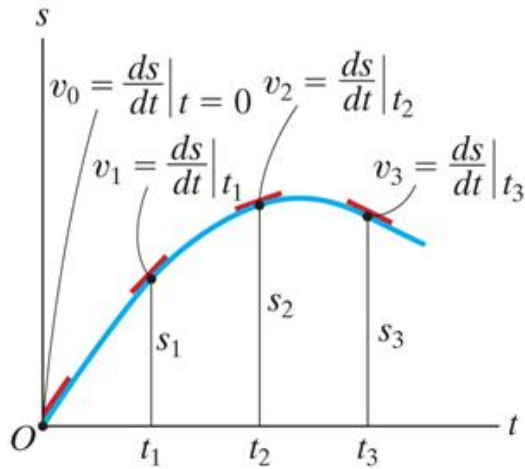
Graphing provides a good way to handle complex motions that would be difficult to describe with formulas.

Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics.

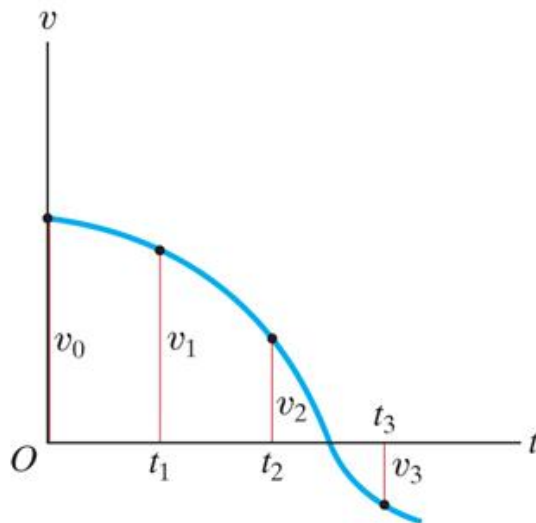
The approach builds on the facts that slope and differentiation are linked and that integration can be thought of as finding the area under a curve.



# s-t Graph



Plots of position vs. time can be used to find velocity vs. time curves. Finding the **slope** of the line tangent to the motion curve at any point is the **velocity** at that point (or  $v = ds/dt$ ).

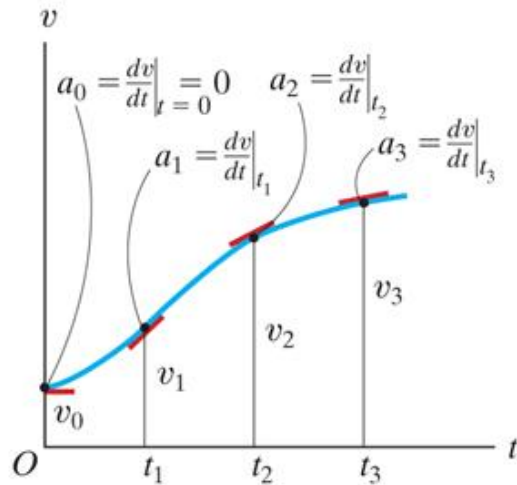


Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.

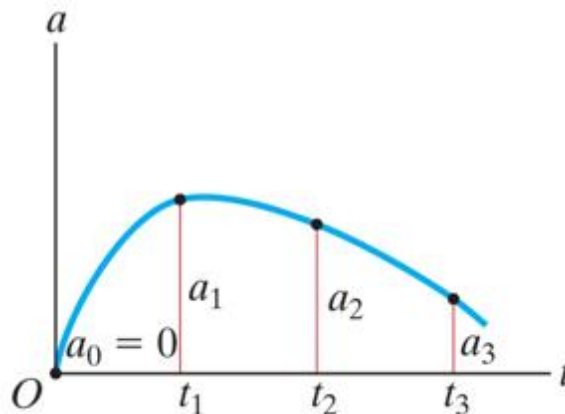




# v-t Graph



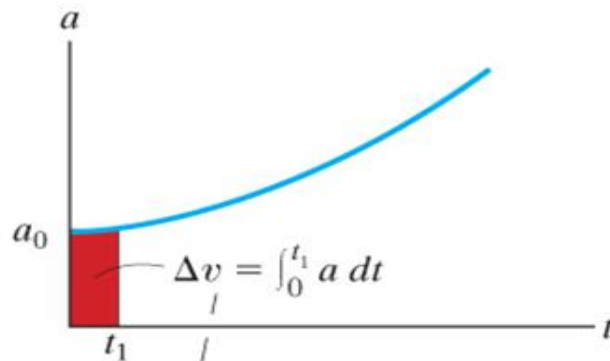
Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the **slope** of the line tangent to the velocity curve at any point is the **acceleration** at that point (or  $a = dv/dt$ ).



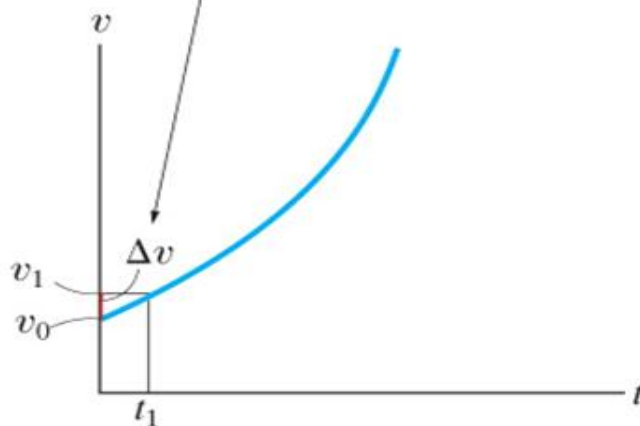
Therefore, the acceleration vs. time (or a-t) graph can be constructed by finding the slope at various points along the v-t graph.

Also, the distance moved (displacement) of the particle is the area under the v-t graph during time  $\Delta t$ .

# a-t Graph



(a)



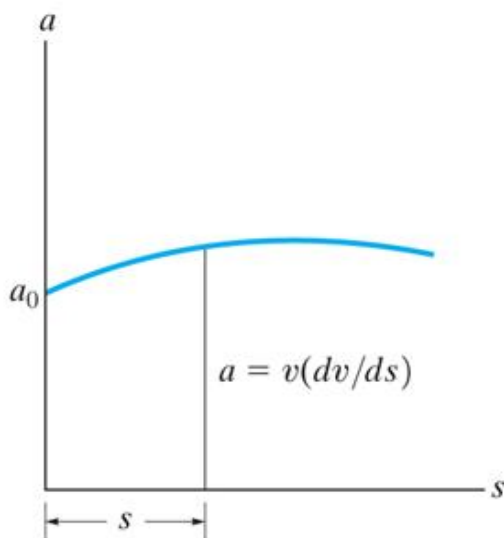
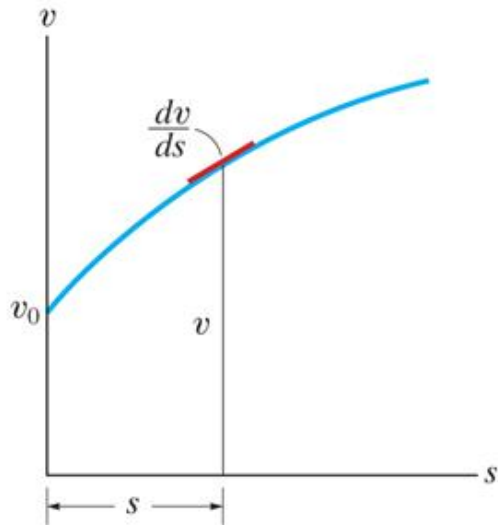
(b)

Given the acceleration vs. time or a-t curve, the change in velocity ( $\Delta v$ ) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle.



# v-s Graph



Another complex case is presented by the velocity vs. distance or v-s graph. By reading the velocity  $v$  at a point on the curve and multiplying it by the slope of the curve ( $dv/ds$ ) at this same point, we can obtain the acceleration at that point.

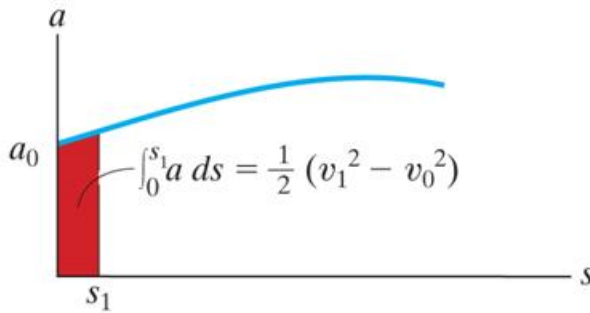
Recall the formula

$$a = v (dv/ds).$$

Thus, we can obtain an a-s plot from the v-s curve.

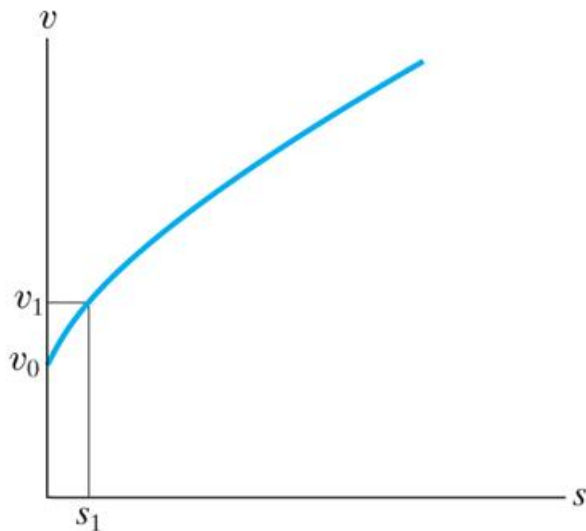


# a-s graph



A more complex case is presented by the acceleration versus position or a-s graph. The area under the a-s curve represents **the change in velocity** (recall  $\int a \, ds = \int v \, dv$ ).

$\frac{1}{2} (v_1^2 - v_0^2) = \int_{s_1}^{s_2} a \, ds =$  area under the  
a-s graph



This equation can be solved for  $v_1$ , allowing you to solve for the velocity at a point. By doing this repeatedly, you can **create a plot of velocity versus distance**.



# Questions & Examples

# Learning Catalytics™

- Please sign in:
  - [www.learningcatalytics.com](http://www.learningcatalytics.com)