CEE 271 APPLIED MECHANICS II

Lecture 27: Equation of Motion - General Plane Motion

Department of Civil & Environmental Engineering
University of Hawaiʻi at Mānoa

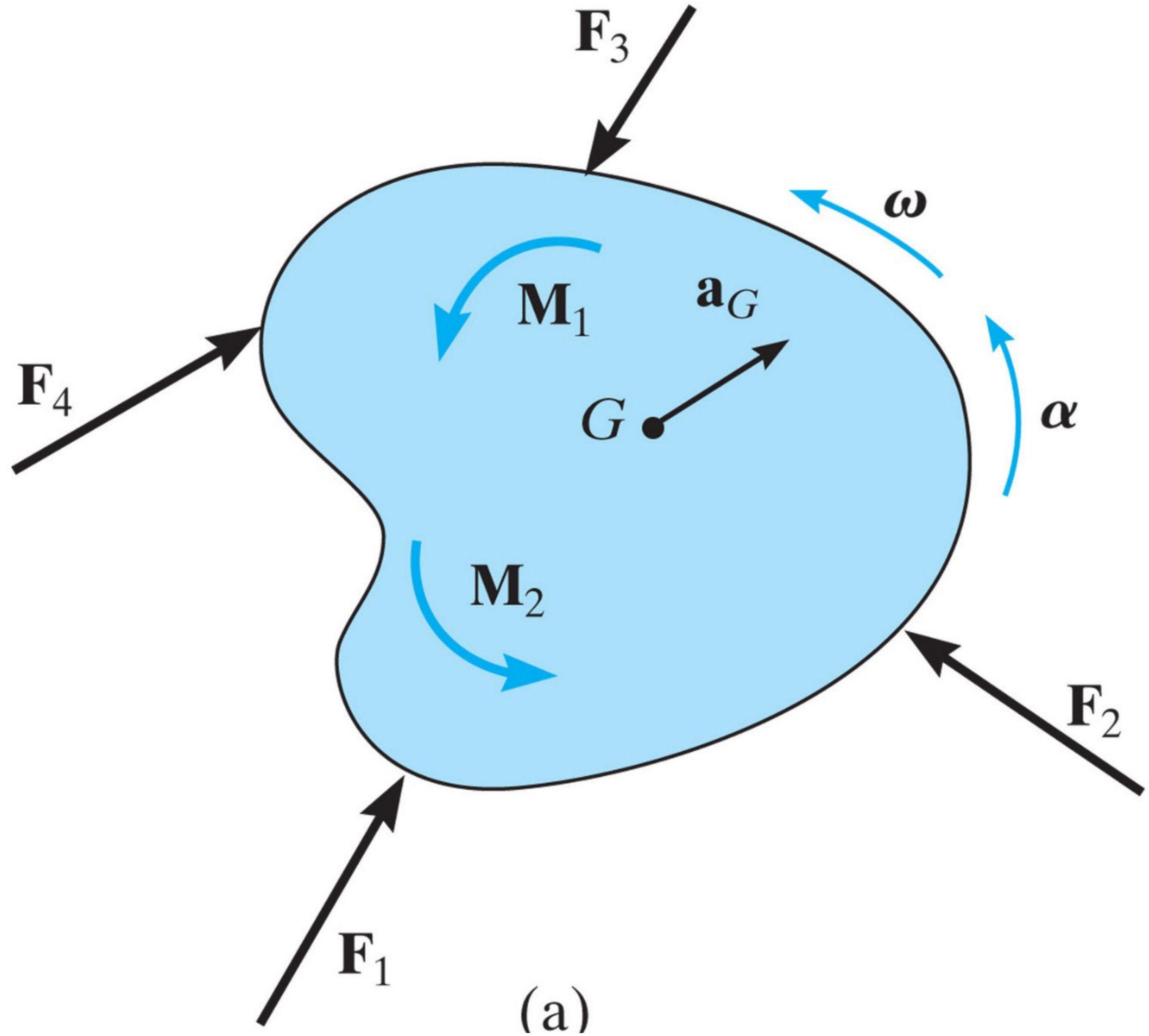
Today's Objectives

- Analyze the planar kinetics of a rigid body undergoing general plane motion.

Outline (Pre-Job Brief)

- Equations of Motion
- Frictional Rolling Problems
- Examples and Questions
- Summary and Feedback

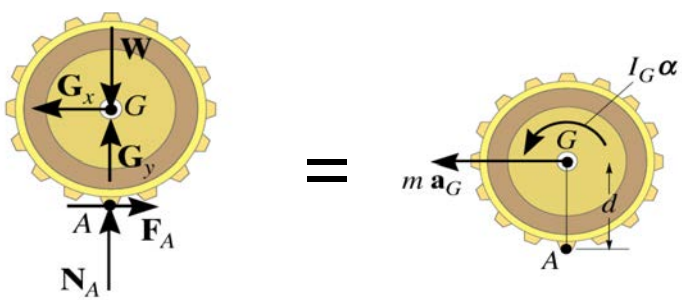
EOM: General Plane Motion



Applications

As the soil compactor accelerates forward, the front roller experiences general plane motion (both translation and rotation).

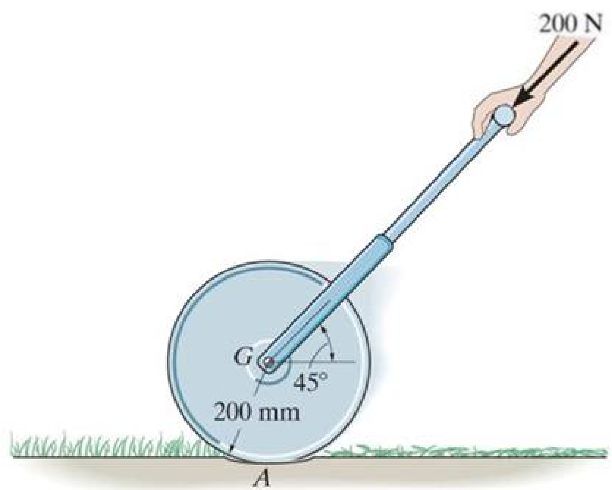
How would you find the loads experienced by the roller shaft or its bearings?



The forces shown on the roller shaft's FBD cause the accelerations shown on the kinetic diagram.

Is point A the IC?

Applications (continued)

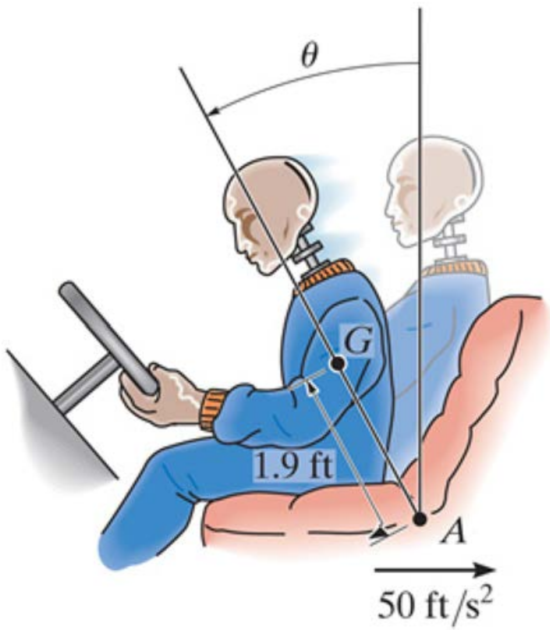


The lawn roller is pushed forward with a force of 200 N when the handle is held at 45°.

How can we determine its translational acceleration and angular acceleration?

Does the total acceleration depend on the coefficient's of static and kinetic friction?

Applications (continued)

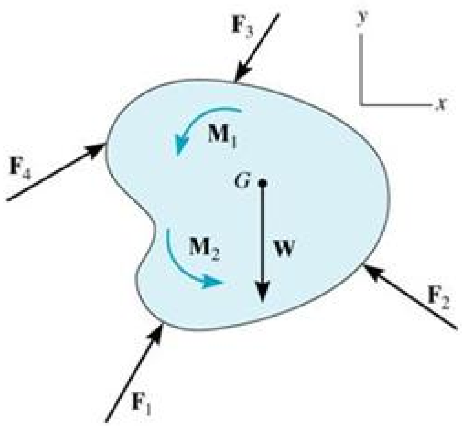


During an impact, the center of gravity G of this crash dummy will decelerate with the vehicle, but also experience another acceleration due to its rotation about point A.

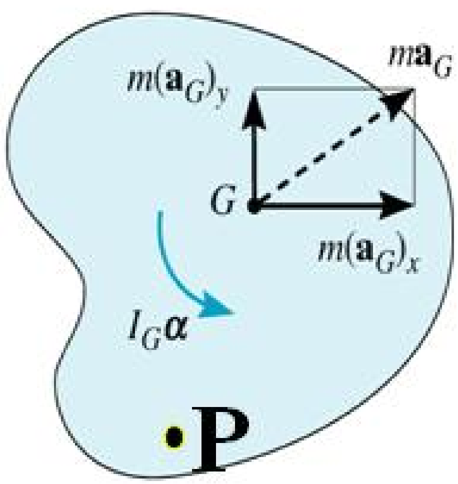
Why?

How can engineers use this information to determine the forces exerted by the seat belt on a passenger during a crash? How would these accelerations impact the design of the seat belt itself?

EOM: General Plane Motion



When a rigid body is subjected to external forces and couple-moments, it can undergo both translational motion and rotational motion. This combination is called general plane motion.



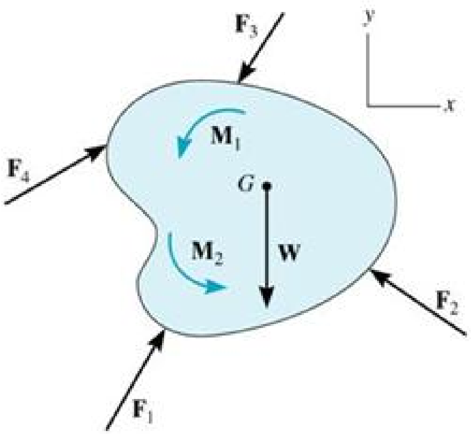
Using an x-y inertial coordinate system, the scalar equations of motions about the center of mass, G, may be written as:

$$\sum F_x = m\,(a_G)_x$$

$$\sum F_y = m\,(a_G)_y$$

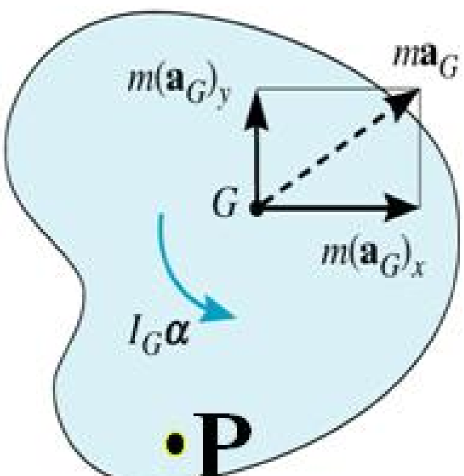
$$\sum M_G = I_G\,\alpha$$

EOM: General Plane Motion



Sometimes, it may be convenient to write the moment equation about a point P, rather than G. Then the equations of motion are written as follows:

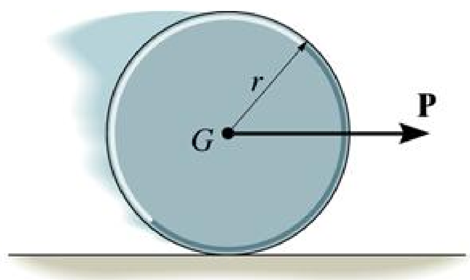
$$\sum F_x = m\,(a_G)_x$$
$$\sum F_y = m\,(a_G)_y$$
$$\sum M_P = \sum (M_k)_P$$



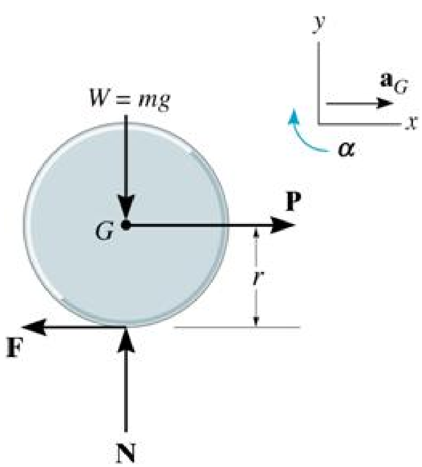
In this case, $\sum (M_k)_P$ represents the sum of the moments of $I_G\alpha$ and $m\mathbf{a}_G$ about point P.

Frictional Rolling Problems

When analyzing the rolling motion of wheels, cylinders, or disks, it may not be known if the body rolls without slipping or if it slips/slides as it rolls.



For example, consider a disk with mass m and radius r, subjected to a known force P.



The equations of motion will be:

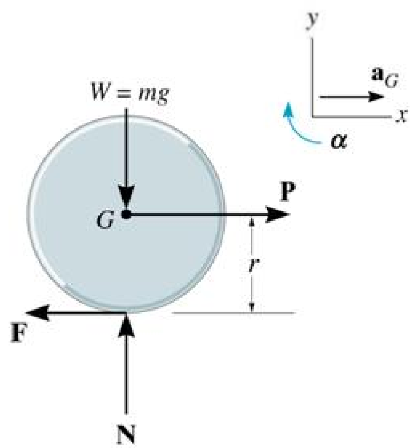
$$\sum F_x = m(a_G)_x \quad \Rightarrow \quad P - F = m\, a_G$$

$$\sum F_y = m(a_G)_y \quad \Rightarrow \quad N - mg = 0$$

$$\sum M_G = I_G \alpha \quad \Rightarrow \quad F\, r = I_G\, \alpha$$

There are 4 unknowns (F, N, α, and a_G) in these three equations.

Frictional Rolling Problems



Hence, we have to make an assumption to provide another equation. Then, we can solve for the unknowns.

The 4th equation can be obtained from the slip or non-slip condition of the disk.

Case 1:

Assume no slipping and use $a_G = \alpha r$ as the 4th equation and DO NOT use $F_f = \mu_s N$. After solving, you will need to verify that the assumption was correct by checking if $F_f \leq \mu_s N$.

Case 2:

Assume slipping and use $F_f = \mu_k N$ as the 4th equation.

In this case, $a_G \neq \alpha r$.

Procedure for Analysis

Problems involving the kinetics of a rigid body undergoing general plane motion can be solved using the following procedure.

1. Establish the x-y inertial coordinate system. Draw both the free body diagram and kinetic diagram for the body.

2. Specify the direction and sense of the acceleration of the mass center, $\mathbf{a}_G$, and the angular acceleration α of the body. If necessary, compute the body's mass moment of inertia I_G.

3. If the moment equation $\Sigma M_p = \Sigma(M_k)_p$ is used, use the kinetic diagram to help visualize the moments developed by the components $m(a_G)_x$, $m(a_G)_y$, and $I_G\alpha$.

4. Apply the three equations of motion.

Procedure for Analysis

5. Identify the unknowns. If necessary (i.e., there are four unknowns), make your slip-no slip assumption (typically no slipping, or the use of $a_G = \alpha\, r$, is assumed first).
6. Use kinematic equations as necessary to complete the solution.
7. If a slip-no slip assumption was made, check its validity!!!

Key points to consider:

1. Be consistent in using the assumed directions. The direction of $\mathbf{a}_G$ must be consistent with α.
2. If $F_f = \mu_k N$ is used, F_f must oppose the motion. As a test, assume no friction and observe the resulting motion. This may help visualize the correct direction of F_f.

Examples & Questions

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