CEE 271 APPLIED MECHANICS II

Lecture 26: Equation of Motion

Translation and Rotation About a Fixed Axis

Department of Civil & Environmental Engineering

University of Hawaiʻi at Mānoa

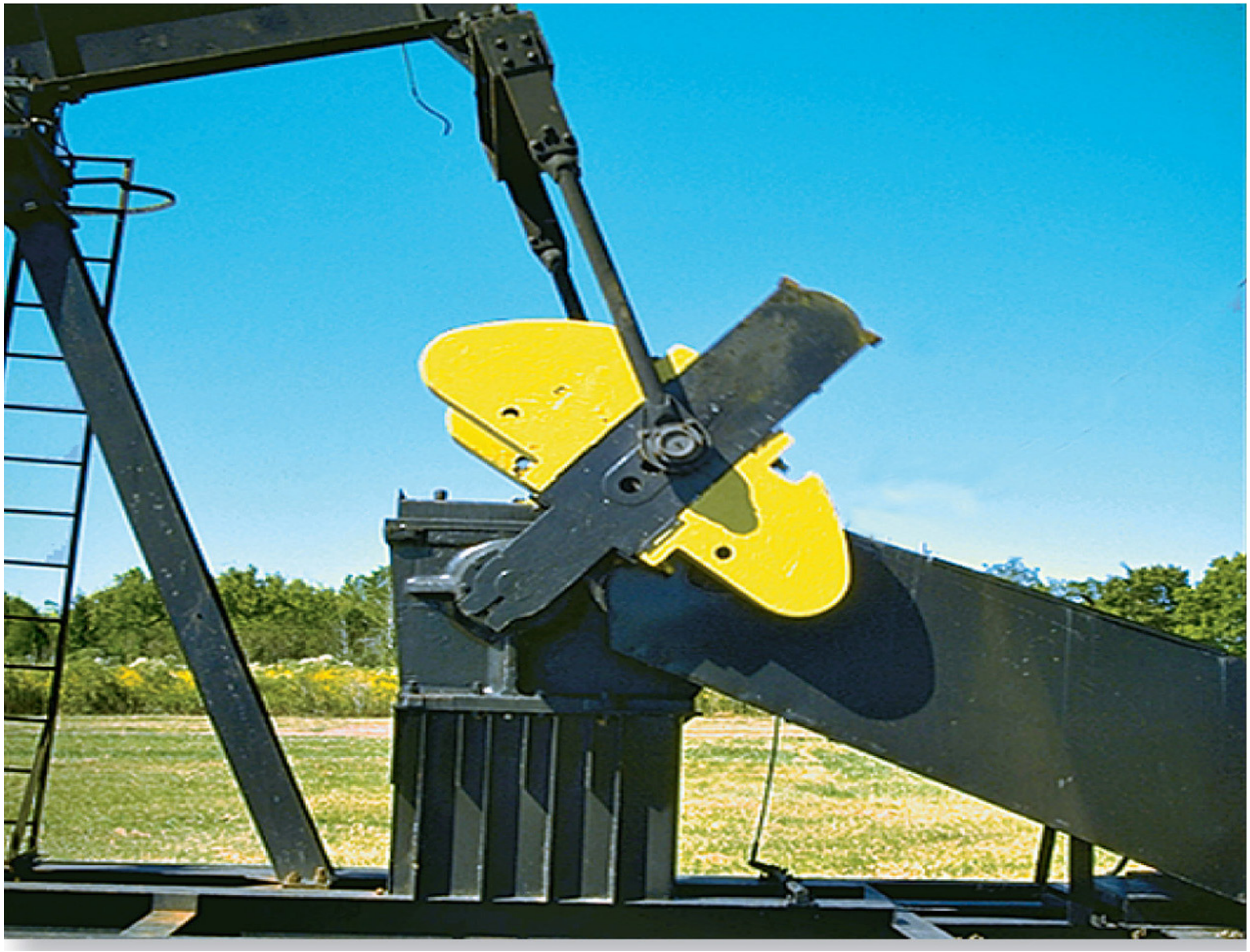
Today's Objectives

- Apply the three equations of motion for a rigid body in planar motion.
- Analyze problems involving translational motion.
- Analyze the planar kinetics of a rigid body undergoing rotational motion.

Outline (Pre-Job Brief)

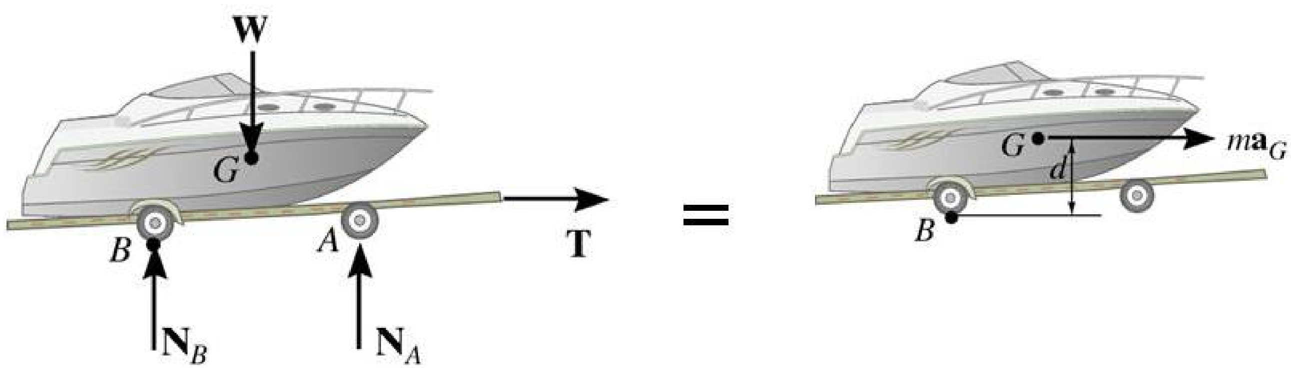
- FBD of Rigid Bodies
- EOM for Rigid Bodies
- Translational Motion
- Rotation about an Axis
- Equations of Motion
- Examples and Questions
- Summary and Feedback

EOM: Translation & Rotation about a fixed axis



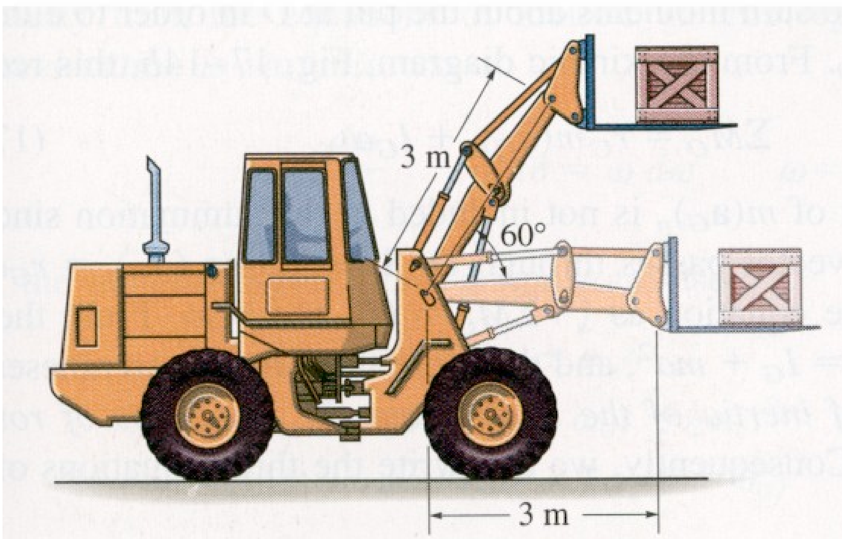
Applications

The boat and trailer undergo rectilinear motion. In order to find the reactions at the trailer wheels and the acceleration of the boat, we need to draw the FBD and kinetic diagram for the boat and trailer.



How many equations of motion do we need to solve this problem? What are they?

Applications (continued)

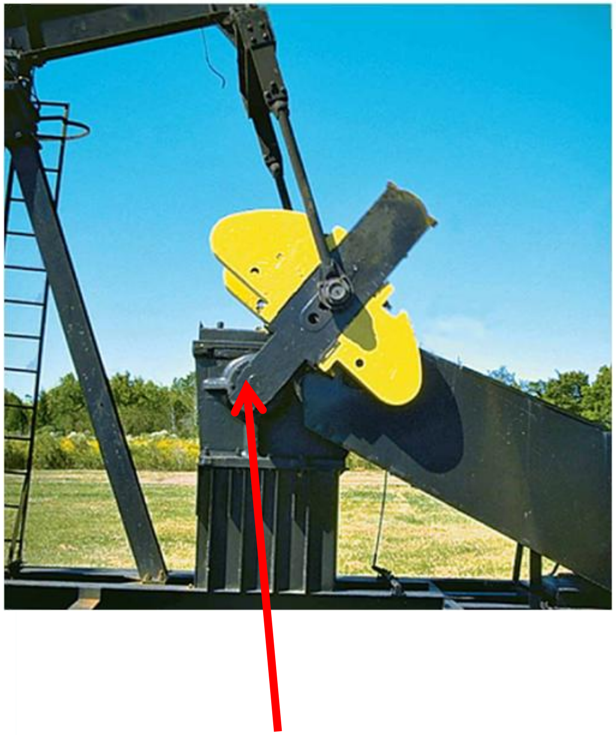


As the tractor raises the load, the crate will undergo curvilinear translation if the forks do not rotate.

If the load is raised too quickly, will the crate slide to the left or right?

How fast can we raise the load before the crate will slide?

Applications (continued)



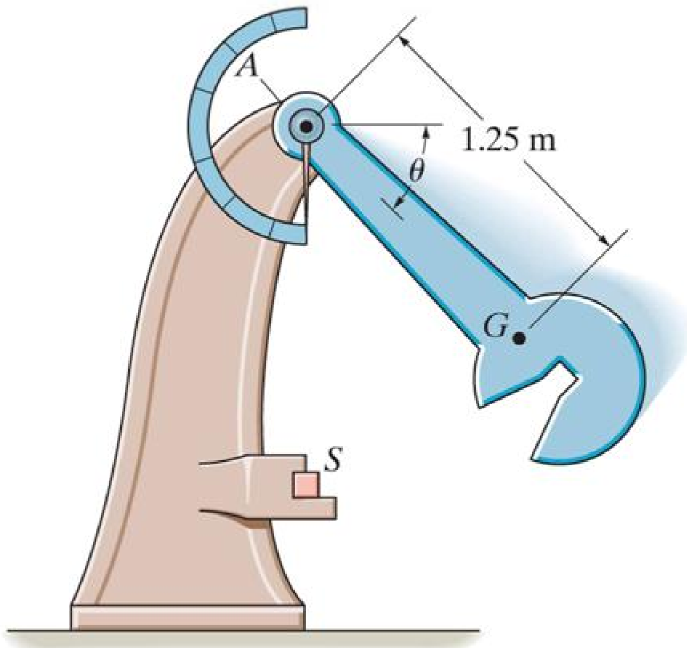
Pin at the center of rotation.

The crank on the oil-pump rig undergoes rotation about a fixed axis, caused by the driving torque, M, from a motor.

As the crank turns, a dynamic reaction is produced at the pin. This reaction is a function of angular velocity, angular acceleration, and the orientation of the crank.

If the motor exerts a constant torque M on the crank, does the crank turn at a constant angular velocity? Is this desirable for such a machine?

Applications (continued)



The pendulum of the Charpy impact machine is released from rest when $\theta = 0°$. Its angular velocity (ω) begins to increase.

Can we determine the angular velocity when it is in vertical position?

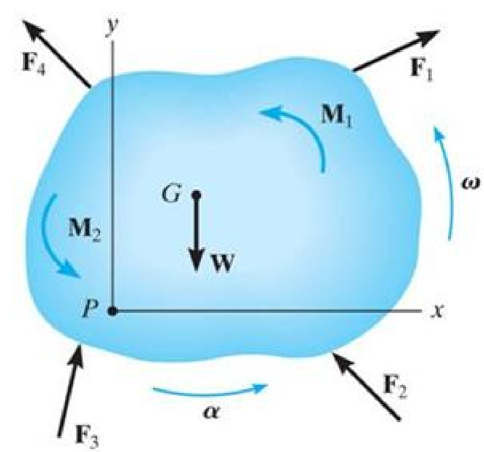
On which property (P) of the pendulum does the angular acceleration (α) depend?

What is the relationship between P and α?

Planar Kinetic EOM

- We will limit our study of planar kinetics to rigid bodies that are symmetric with respect to a fixed reference plane.
- As discussed in Chapter 16, when a body is subjected to general plane motion, it undergoes a combination of translation and rotation.
- First, a coordinate system with its origin at an arbitrary point P is established.

  The x-y axes should not rotate but can either be fixed or translate with constant velocity.

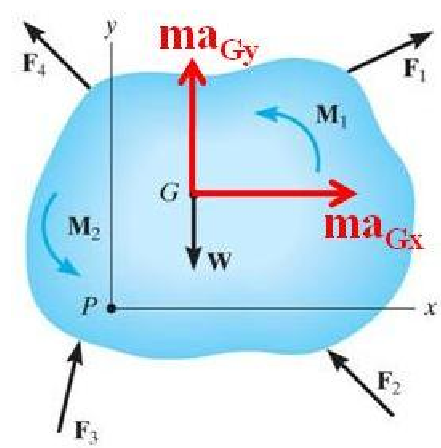


EOM: Translational

- If a body undergoes translational motion, the equation of motion is $\Sigma \boldsymbol{F} = m\,\mathbf{a}_G$. This can also be written in scalar form as

$$\Sigma F_x = m(a_G)_x \quad \text{and} \quad \Sigma F_y = m(a_G)_y$$

- In words: the sum of all the external forces acting on the body is equal to the body's mass times the acceleration of it's mass center.



EOM: Rotational

We need to determine the effects caused by the moments of an external force system.

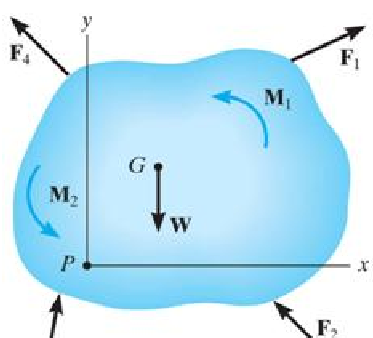
The moment about point P can be written as:

$$\Sigma\,(\boldsymbol{r}_{\mathrm{i}} \times \boldsymbol{F}_{\mathrm{i}}) + \Sigma\,\boldsymbol{M}_{\mathrm{i}} = \bar{\boldsymbol{r}} \times m\mathbf{a}_{G} + I_{G}\boldsymbol{\alpha}$$

$$\text{and}\ \ \Sigma\, M_{p} = \Sigma(M_{k})_{p}$$

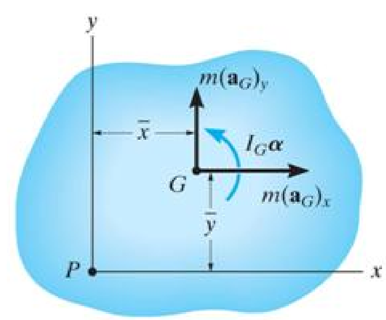
where $\bar{\boldsymbol{r}} = \bar{x}\,\boldsymbol{i} + \bar{y}\,\boldsymbol{j}$ and $\Sigma\, M_p$ is the resultant moment about P due to all the external forces.

The term $\Sigma(M_k)_{\mathbf{p}}$ is called the kinetic moment about point P.



Free-body diagram

=



Kinetic diagram

EOM: Rotational

If point P coincides with the mass center G, this equation reduces to the scalar equation of $\Sigma M_G = I_G \alpha$.

In words: the resultant (summation) moment about the mass center due to all the external forces is equal to the moment of inertia about G times the angular acceleration of the body.

Thus, three independent scalar equations of motion may be used to describe the general planar motion of a rigid body. These equations are:

$$\Sigma F_x = m(a_G)_x$$

$$\Sigma F_y = m(a_G)_y$$

and $\Sigma M_G = I_G\alpha$ or $\Sigma M_p = \Sigma (M_k)_p$

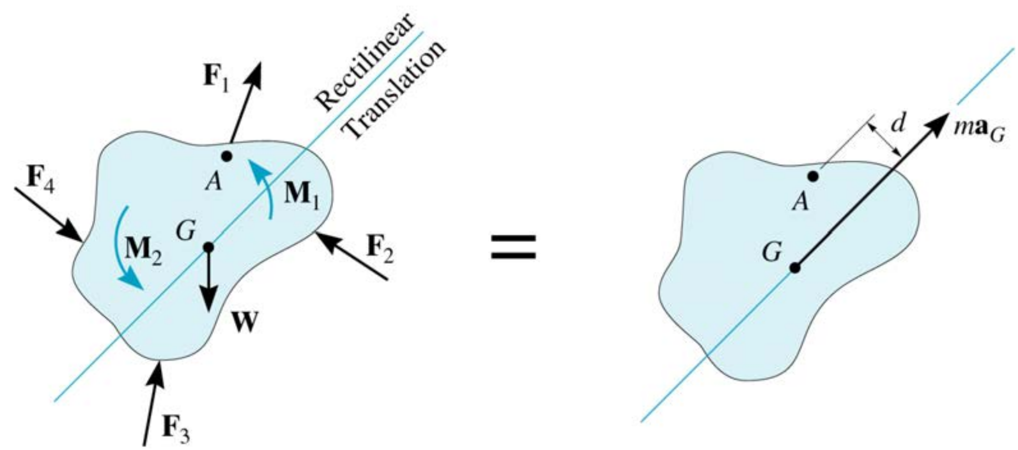
EOM: Translation

When a rigid body undergoes only translation, all the particles of the body have the same acceleration so $\mathbf{a}_G = \mathbf{a}$ and $\alpha = 0$. The equations of motion become:

$$\Sigma F_x = m(a_G)_x$$

$$\Sigma F_y = m(a_G)_y$$

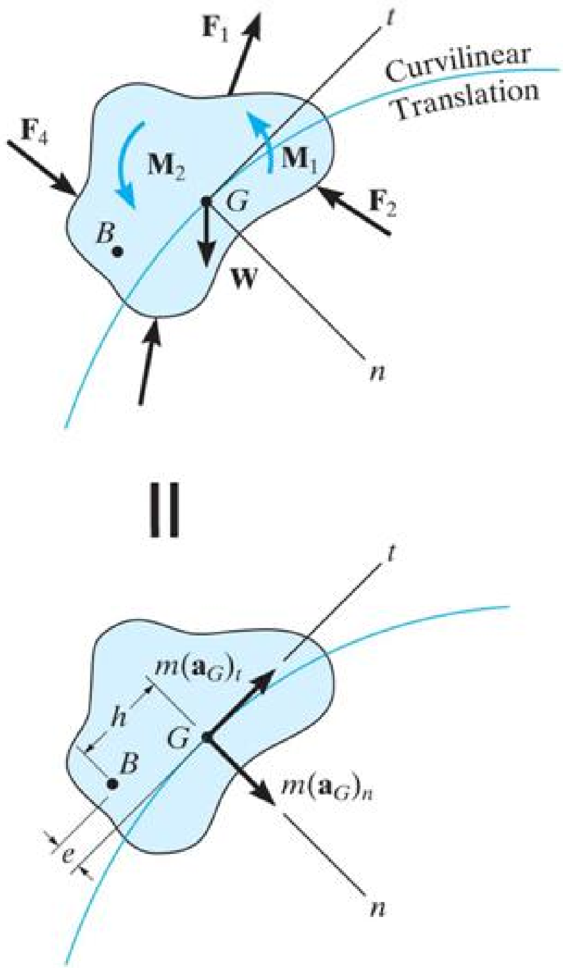
$$\Sigma M_G = 0$$



Note that, if it makes the problem easier, the moment equation can be applied about another point instead of the mass center. For example, if point A is chosen,

$$\Sigma M_A = (m\, a_G)\, d\,.$$

EOM: Translation



When a rigid body is subjected to curvilinear translation, it is best to use an n-t coordinate system.

Then apply the equations of motion, as written below, for n-t coordinates.

$$\Sigma F_n = m(a_G)_n$$

$$\Sigma F_t = m(a_G)_t$$

$$\Sigma M_G = 0 \quad \text{or}$$

$$\Sigma M_B = e[m(a_G)_t] - h[m(a_G)_n]$$

Procedure for Analysis

Problems involving kinetics of a rigid body in only translation should be solved using the following procedure:

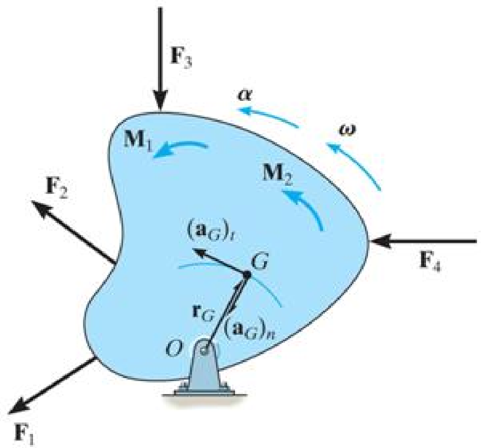
1. Establish an (x-y) or (n-t) inertial coordinate system and specify the sense and direction of acceleration of the mass center, $\mathbf{a}_G$.
2. Draw a FBD and kinetic diagram showing all external forces, couples and the inertia forces and couples.
3. Identify the unknowns.
4. Apply the three equations of motion (one set or the other):

$$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y \qquad \Big| \qquad \Sigma F_n = m(a_G)_n \quad \Sigma F_t = m(a_G)_t$$

$$\Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P \qquad \Big| \qquad \Sigma M_G = 0 \quad \text{or} \quad \Sigma M_P = \Sigma (M_k)_P$$

5. Remember, friction forces always act on the body opposing the motion of the body.

EOM: Fixed Axis



When a rigid body rotates about a fixed axis perpendicular to the plane of the body at point O, the body's center of gravity G moves in a circular path of radius r_G. Thus, the acceleration of point G can be represented by a tangential component $(a_G)_t = r_G\,\alpha$ and a normal component $(a_G)_n = r_G\,\omega^2$.

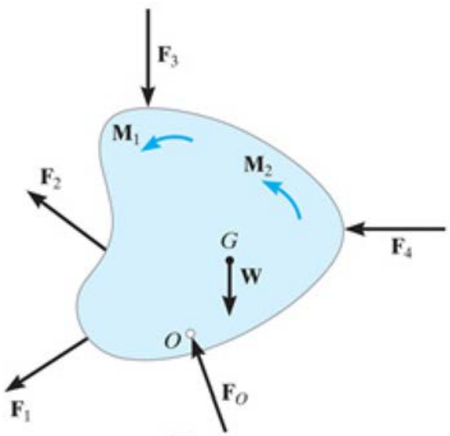
Since the body experiences an angular acceleration, its inertia creates a moment of magnitude, $I_g\alpha$, equal to the moment of the external forces about point G. Thus, the scalar equations of motion can be stated as:

$$\sum F_n = m\,(a_G)_n = m\,r_G\,\omega^2$$
$$\sum F_t = m\,(a_G)_t = m\,r_G\,\alpha$$
$$\sum M_G = I_G\,\alpha$$

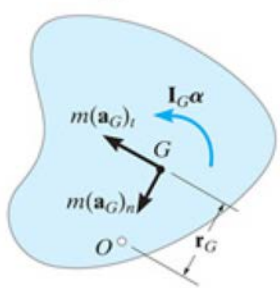
EOM (continued)

Note that the $\sum M_G$ moment equation may be replaced by a moment summation about any arbitrary point. Summing the moment about the center of rotation O yields

$$\sum M_O = I_G \alpha + r_G\, m\, (a_G)_t = [I_G + m(r_G)^2]\, \alpha$$



=



From the parallel axis theorem, $I_O = I_G + m(r_G)^2$, therefore the term in parentheses represents I_O.

Consequently, we can write the three equations of motion for the body as:

$$\sum F_n = m\,(a_G)_n = m\, r_G\, \omega^2$$

$$\sum F_t = m\,(a_G)_t = m\, r_G\, \alpha$$

$$\sum M_O = I_O\, \alpha$$

Procedure for Analysis

Problems involving the kinetics of a rigid body rotating about a fixed axis can be solved using the following process.

1. Establish an inertial coordinate system and specify the sign and direction of $(a_G)_n$ and $(a_G)_t$.

2. Draw a free body diagram accounting for all external forces and couples. Show the resulting inertia forces and couple (typically on a separate kinetic diagram).

3. Compute the mass moment of inertia I_G or I_O.

4. Write the three equations of motion and identify the unknowns. Solve for the unknowns.

5. Use kinematics if there are more than three unknowns (since the equations of motion allow for only three unknowns).

Examples & Questions

Learning Catalytics™

- Please sign in:
  - www.learningcatalytics.com