CEE 271 APPLIED MECHANICS II

Lecture 25: Moment of Inertia

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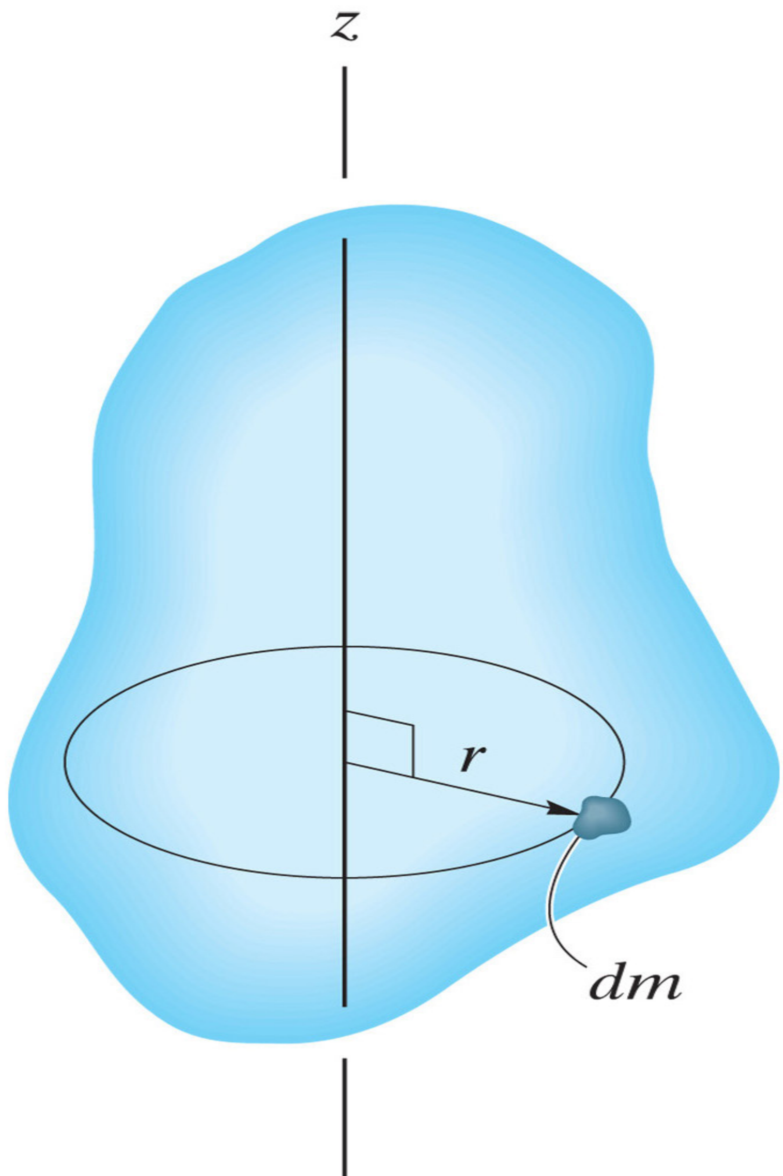
Today's Objectives

- Determine the mass moment of inertia of a rigid body or a system of rigid bodies.

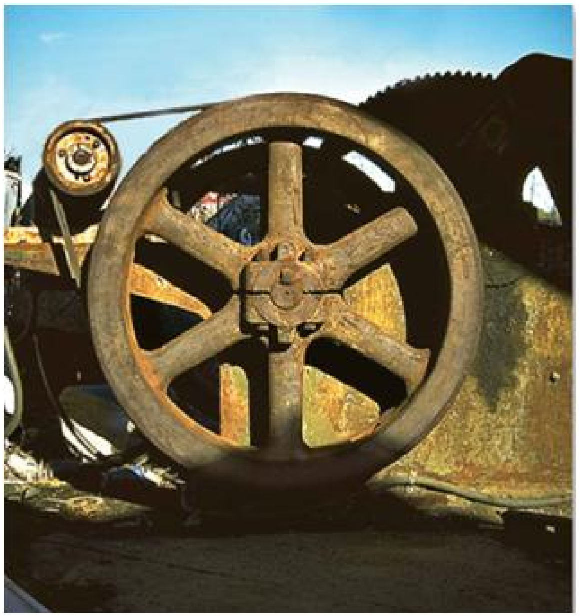
Outline (Pre-Job Brief)

- Mass Moment of Inertia
- Parallel-Axis Theorem
- Composite Bodies
- Examples and Questions
- Summary and Feedback

Mass Moment of Inertia



Applications



The large flywheel in the picture is connected to a large metal cutter. The flywheel mass is used to help provide a uniform motion to the cutting blade.

What property of the flywheel is most important for this use? How can we determine a value for this property?

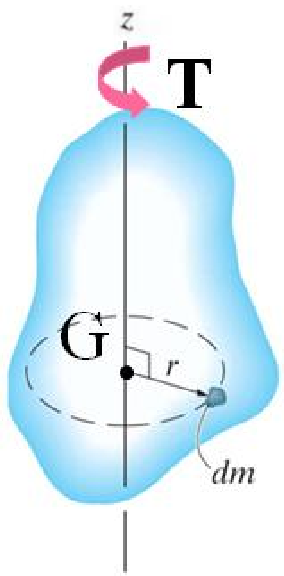
Why is most of the mass of the flywheel located near the flywheel's circumference?

Applications (continued)

The crank on the oil-pump rig undergoes rotation about a fixed axis that is not at its mass center. The crank develops a kinetic energy directly related to its mass moment of inertia. As the crank rotates, its kinetic energy is converted to potential energy and vice versa.

Is the mass moment of inertia of the crank about its axis of rotation smaller or larger than its moment of inertia about its center of mass?

Mass Moment of Inertia

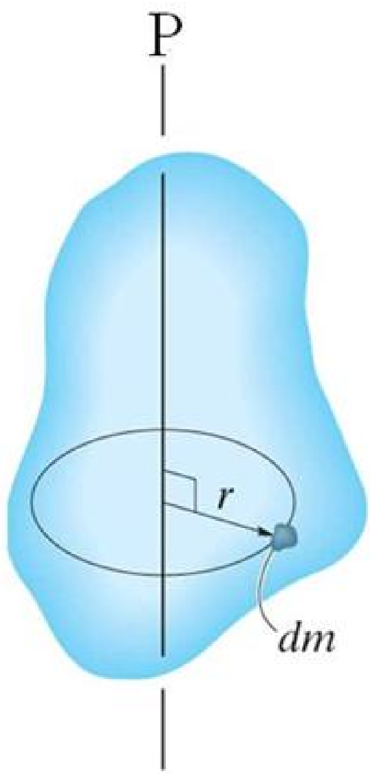


Consider a rigid body with a center of mass at G. It is free to rotate about the z axis, which passes through G. Now, if we apply a torque T about the z axis to the body, the body begins to rotate with an angular acceleration of α.

T and α are related by the equation $T = I\,\alpha$. In this equation, I is the mass moment of inertia (MMI) about the z axis.

The MMI of a body is a property that measures the resistance of the body to angular acceleration. The MMI is often used when analyzing rotational motion.

Mass Moment of Inertia

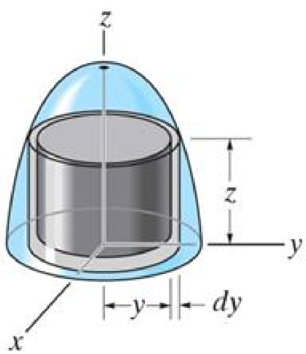


Consider a rigid body and the arbitrary axis P shown in the figure. The MMI about the P axis is defined as $I = \int_m r^2 \, dm$, where r, the "moment arm," is the perpendicular distance from the axis to the arbitrary element dm.

The mass moment of inertia is always a positive quantity and has a unit of $kg \cdot m^2$ or $slug \cdot ft^2$.

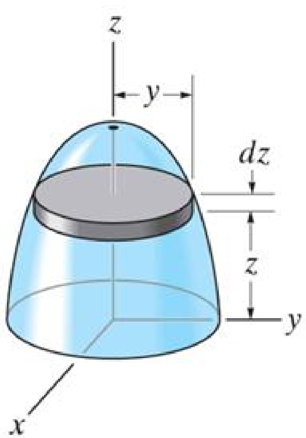
Procedure for Analysis

When using direct integration, only symmetric bodies having surfaces generated by revolving a curve about an axis will be considered here.



Shell element

- If a shell element having a height z, radius r = y, and thickness dy is chosen for integration, then the volume element is $dV = (2\pi y)(z)dy$.
- This element may be used to find the moment of inertia I_z since the entire element, due to its thinness, lies at the same perpendicular distance y from the z-axis.

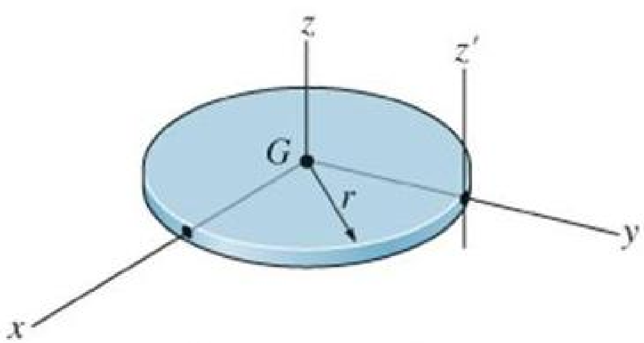


Disk element

- If a disk element having a radius y and a thickness dz is chosen for integration, then the volume $dV = (\pi y^2)dz$.
- Using the moment of inertia of the disk element, we can integrate to determine the moment of inertia of the entire body.

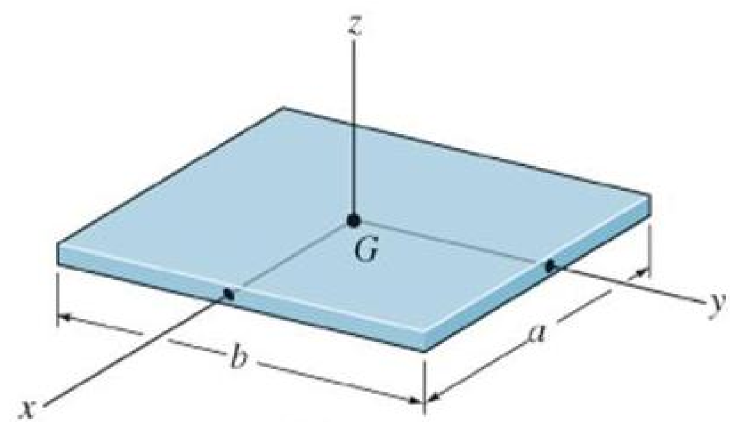
Mass Moment of Inertia

The figures below show the mass moment of inertia formulations for two shapes commonly used when working with three dimensional bodies. These shapes are often used as the differential element being integrated over an entire body.



Thin Circular disk

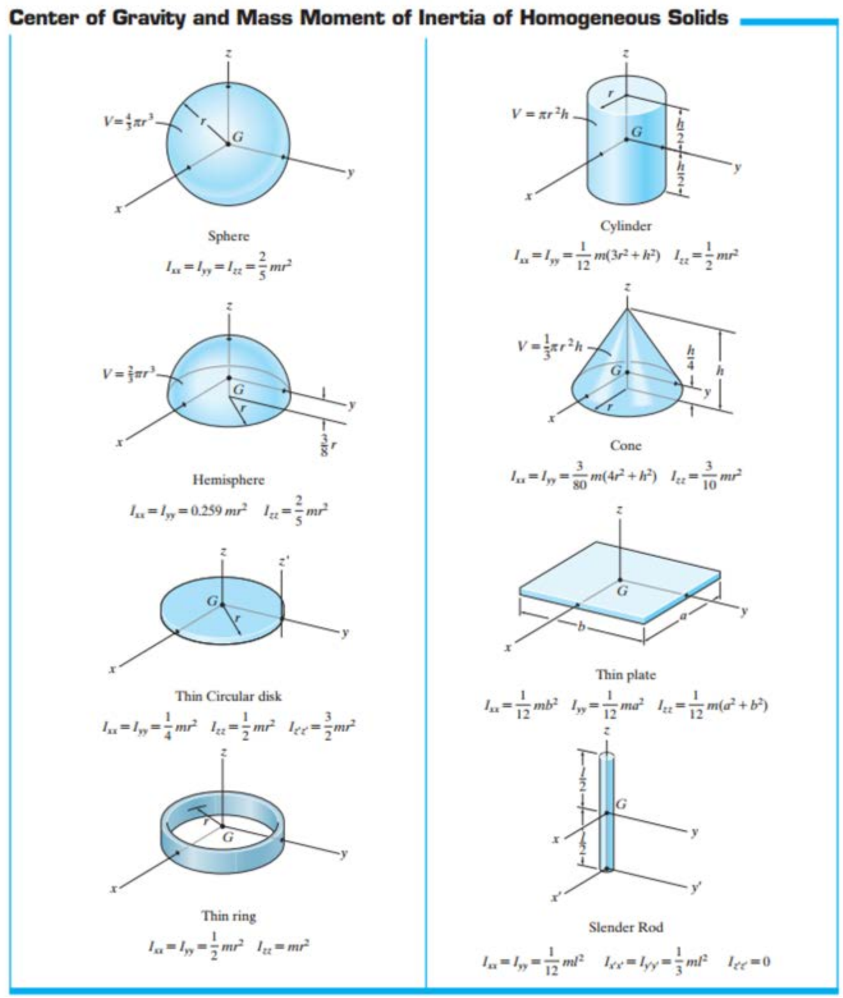
$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2}mr^2$



Thin plate

$I_{xx} = \frac{1}{12}mb^2 \quad I_{yy} = \frac{1}{12}ma^2 \quad I_{zz} = \frac{1}{12}m(a^2 + b^2)$

Mass Moment of Inertia



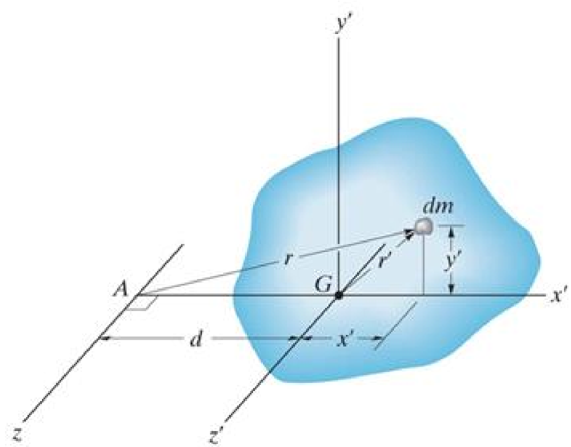
Additional standard shapes are given on the back cover of textbook.

Parallel-Axis Theorem

If the mass moment of inertia of a body about an axis passing through the body's mass center is known, then the moment of inertia about any other parallel axis may be determined by using the parallel axis theorem,

$$I = I_G + md^2$$

where I_G = mass moment of inertia about the body's mass center
m = mass of the body
d = perpendicular distance between the parallel axes



Radius of Gyration & Composite Bodies

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Radius of Gyration

The mass moment of inertia of a body about a specific axis can be defined using the radius of gyration (k). The radius of gyration has units of length and is a measure of the distribution of the body's mass about the axis at which the moment of inertia is defined.

$$I = m\,k^2 \quad \text{or} \quad k = \sqrt{(I/m)}$$

Composite Bodies

If a body is constructed of a number of simple shapes, such as disks, spheres, or rods, the mass moment of inertia of the body about any axis can be determined by algebraically adding together all the mass moments of inertia, found about the same axis, of the different shapes.

Examples & Questions

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