# CEE 271 APPLIED MECHANICS II

## Lecture 24: Relative Acceleration Analysis

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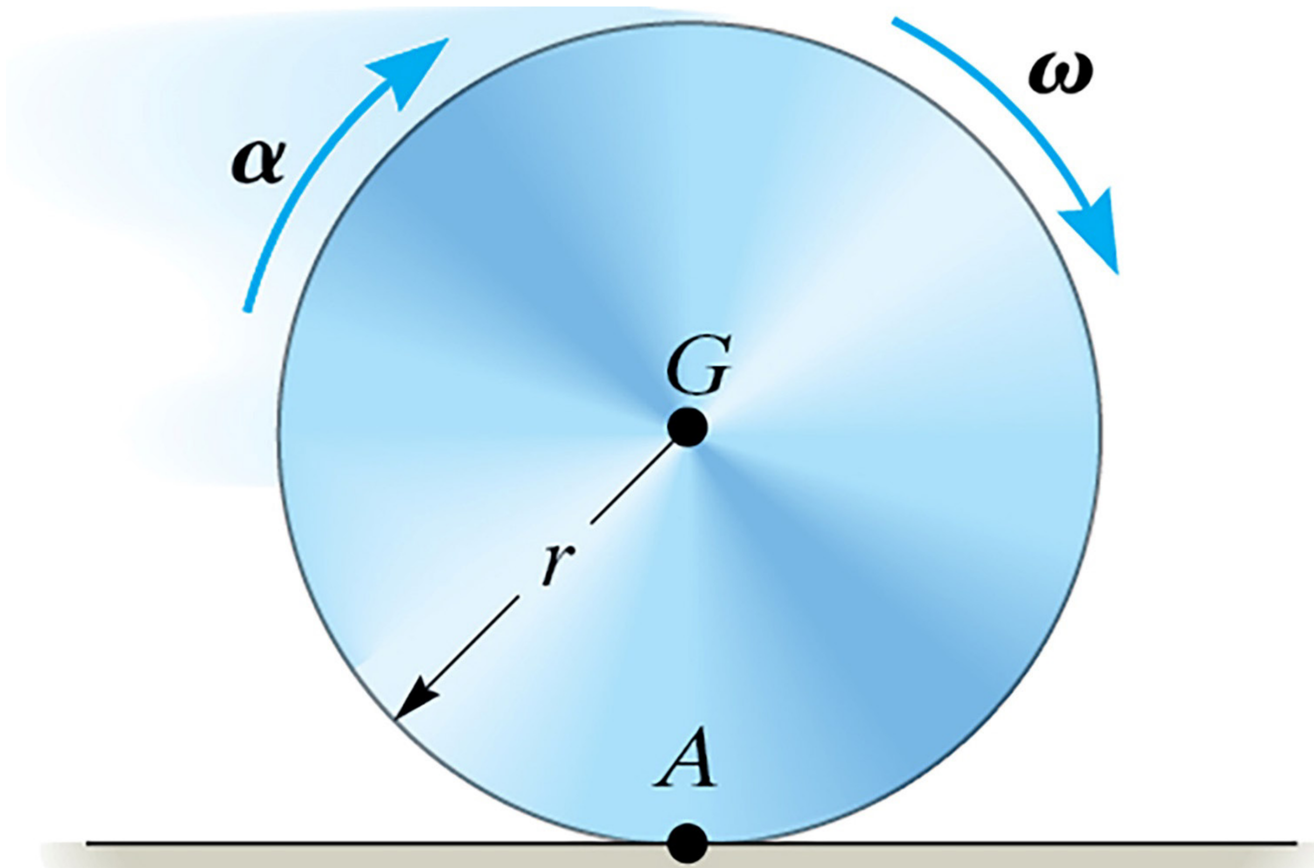
# Today's Objectives

- Resolve the acceleration of a point on a body into components of translation and rotation.
- Determine the acceleration of a point on a body by using a relative acceleration analysis.

# Outline (Pre-Job Brief)

- Translation and Rotation Components of Acceleration
- Relative Acceleration Analysis
- Roll-Without-Slip Motion
- Examples and Questions
- Summary and Feedback

# Relative Acceleration Motion



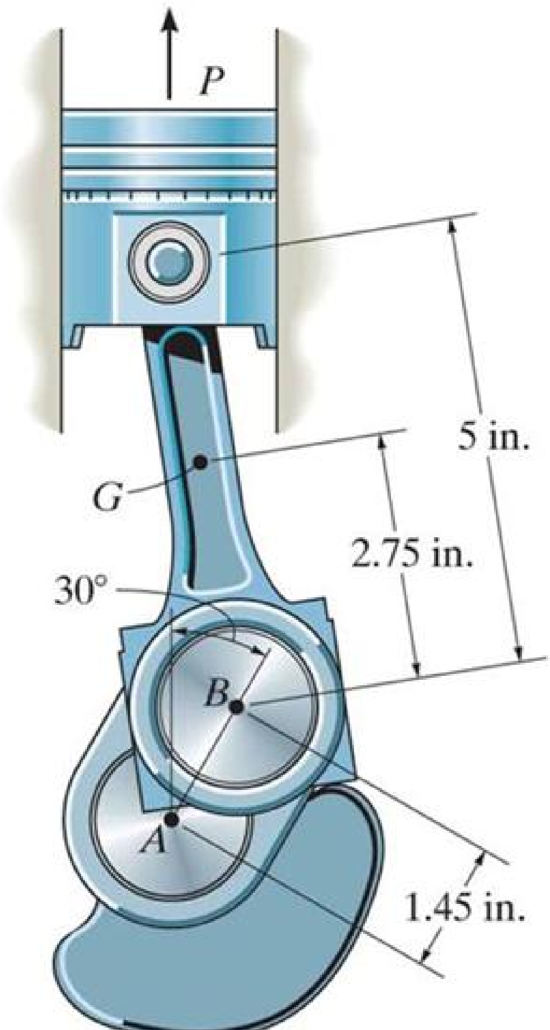
# Applications


In the mechanism for a window, link AC rotates about a fixed axis through C, and AB undergoes general plane motion. Since point A moves along a curved path, it has two components of acceleration while point B, sliding in a straight track, has only one.

The components of acceleration of these points can be inferred since their motions are known.

How can we determine the accelerations of the links in the mechanism?

# Applications (continued)




In an automotive engine, the forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston.

How can we relate the accelerations of the piston, connection rod, and crankshaft to each other?

# Relative Acceleration

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

$$\frac{d\boldsymbol{v}_B}{dt} = \frac{d\boldsymbol{v}_A}{dt} + \frac{d\boldsymbol{v}_{B/A}}{dt}$$

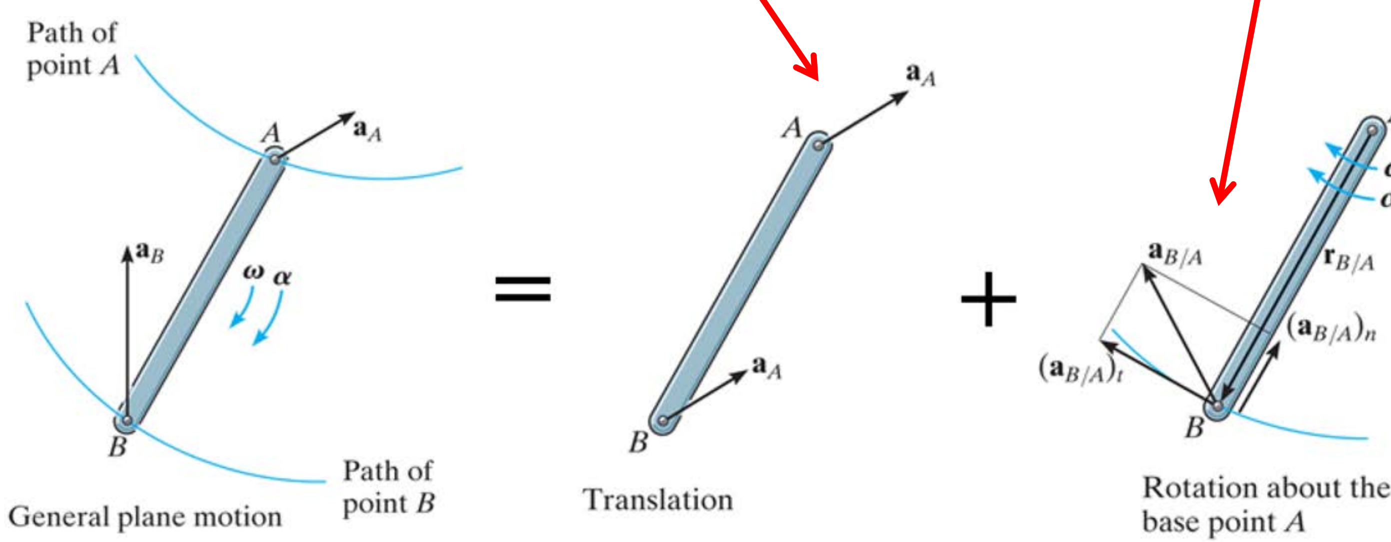
These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A and includes both tangential and normal components.

The result is $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$

# Relative Acceleration

Graphically: $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$




The relative tangential acceleration component $(\mathbf{a}_{B/A})_t$ is $(\alpha \times \boldsymbol{r}_{B/A})$ and perpendicular to $\boldsymbol{r}_{B/A}$.

The relative normal acceleration component $(\mathbf{a}_{B/A})_n$ is $(-\omega^2 \, \boldsymbol{r}_{B/A})$ and the direction is always from B towards A.

# Relative Acceleration

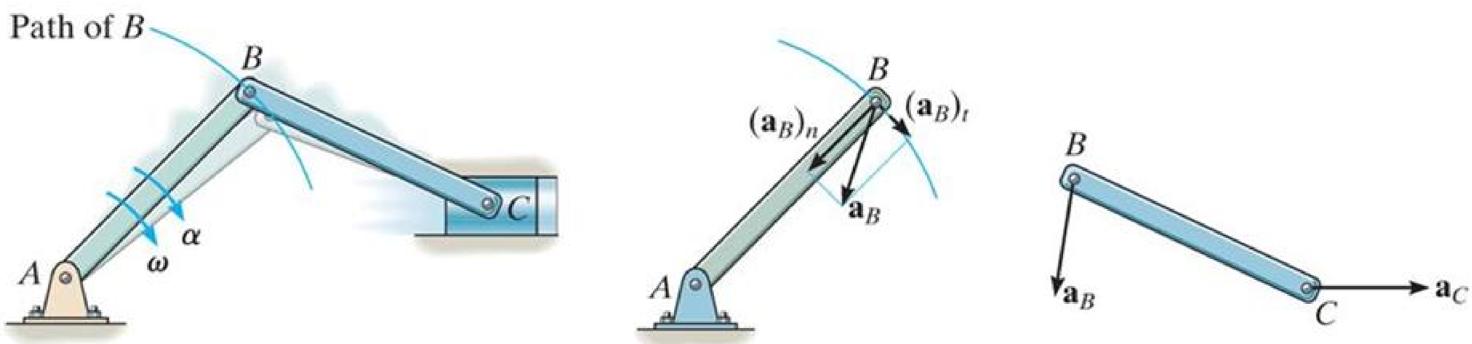
Since the relative acceleration components can be expressed as $(\mathbf{a}_{B/A})_t = \alpha \times \boldsymbol{r}_{B/A}$ and $(\mathbf{a}_{B/A})_n = -\,\omega^2\, \boldsymbol{r}_{B/A}$, the relative acceleration equation becomes

$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \boldsymbol{r}_{B/A} - \omega^2\, \boldsymbol{r}_{B/A}$$

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, $\omega^2$) and the relative position vector, $\boldsymbol{r}_{B/A}$.

# Application

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.




In this mechanism, point B is known to travel along a circular path, so $\mathbf{a}_B$ can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, $\mathbf{a}_C$ is directed horizontally.

# Procedure for Analysis

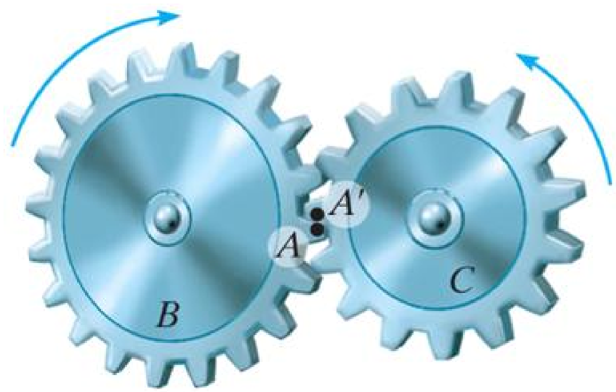
1. Establish a fixed coordinate system.
2. Draw the kinematic diagram of the body.
3. Indicate on it $\mathbf{a}_A$, $\mathbf{a}_B$, $\omega$, $\alpha$, and $\boldsymbol{r}_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $\mathbf{a}_A = (\mathbf{a}_A)_t + (\mathbf{a}_A)_n$ and $\mathbf{a}_B = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n$.
4. Apply the relative acceleration equation:

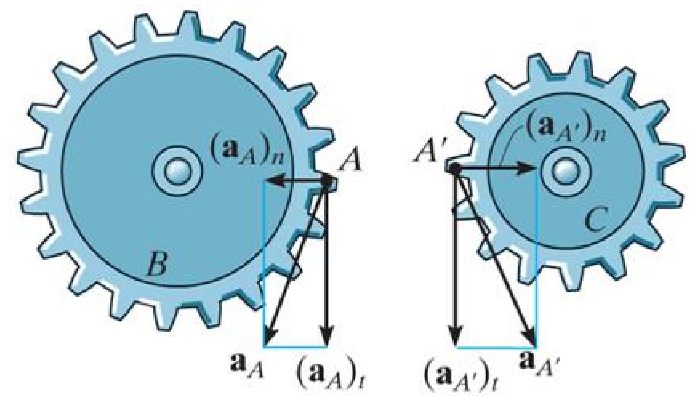
$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \boldsymbol{r}_{B/A} - \omega^2 \boldsymbol{r}_{B/A}$$

5. If the solution yields a negative answer for an unknown magnitude, this indicates that the sense of direction of the vector is opposite to that shown on the diagram.

# Bodies in Contact

Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.




In this case, the tangential components of acceleration will be the same, i. e.,

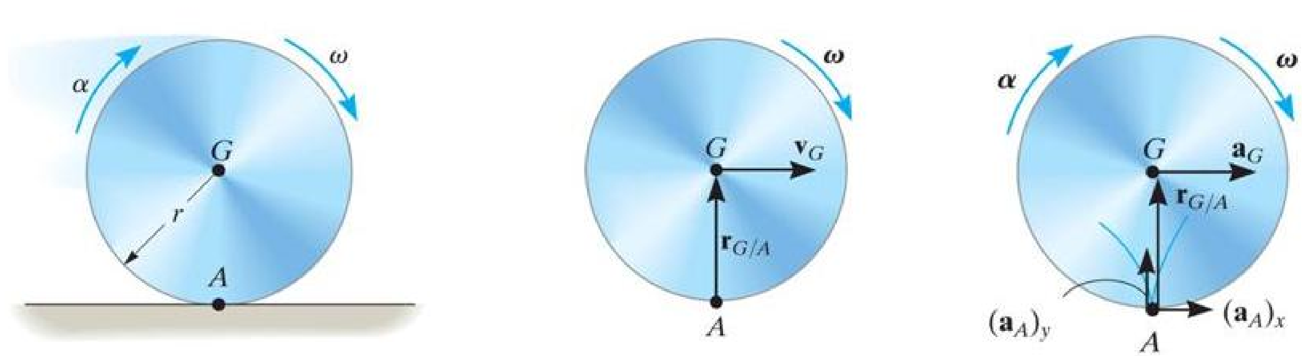
$$(\mathbf{a}_A)_t = (\mathbf{a}_{A'})_t \text{ (which implies } \alpha_B r_B = \alpha_C r_C \text{).}$$

The normal components of acceleration will not be the same.

$$(\mathbf{a}_A)_n \neq (\mathbf{a}_{A'})_n \text{ so } \mathbf{a}_A \neq \mathbf{a}_{A'}$$

# Rolling Motion

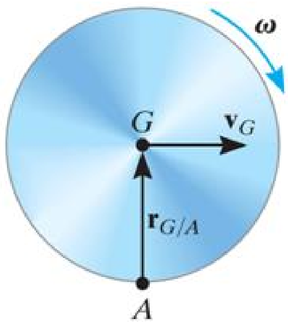
Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball, cylinder, or disk rolling without slipping. This situation can be analyzed using relative velocity and acceleration equations.




As the cylinder rolls, point G (center) moves along a straight line. If ω and α are known, the relative velocity and acceleration equations can be applied to A, at the instant A is in contact with the ground. The point A is the instantaneous center of zero velocity, however it is not a point of zero acceleration.

# Rolling Motion (continued)

- Velocity:



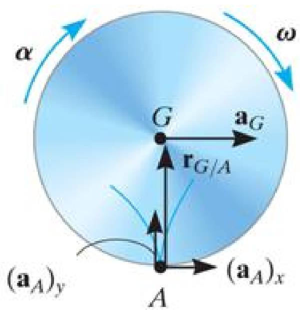

Since no slip occurs, $\boldsymbol{v}_A = \boldsymbol{0}$ when A is in contact with ground. From the kinematic diagram:

$$\boldsymbol{v}_G = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{G/A}$$

$$v_G\,\boldsymbol{i} = \boldsymbol{0} + (-\omega\boldsymbol{k}) \times (r\,\boldsymbol{j})$$

$$v_G = \omega r \quad \text{or} \quad \boldsymbol{v}_G = \omega r\,\boldsymbol{i}$$

- Acceleration:




Since G moves along a straight-line path, $\mathbf{a}_G$ is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \boldsymbol{r}_{G/A} - \omega^2\boldsymbol{r}_{G/A} \;\Rightarrow\; a_G\,\boldsymbol{i} = a_A\,\boldsymbol{j} + (-\alpha\,\boldsymbol{k}) \times (r\,\boldsymbol{j}) - \omega^2(r\,\boldsymbol{j})$$

Evaluating and equating $\boldsymbol{i}$ and $\boldsymbol{j}$ components:

$$a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad \mathbf{a}_G = \alpha r\,\boldsymbol{i} \quad \text{and} \quad \mathbf{a}_A = \omega^2 r\,\boldsymbol{j}$$

# Examples & Questions



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