CEE 271 APPLIED MECHANICS II

Lecture 21: Relative Velocity Analysis

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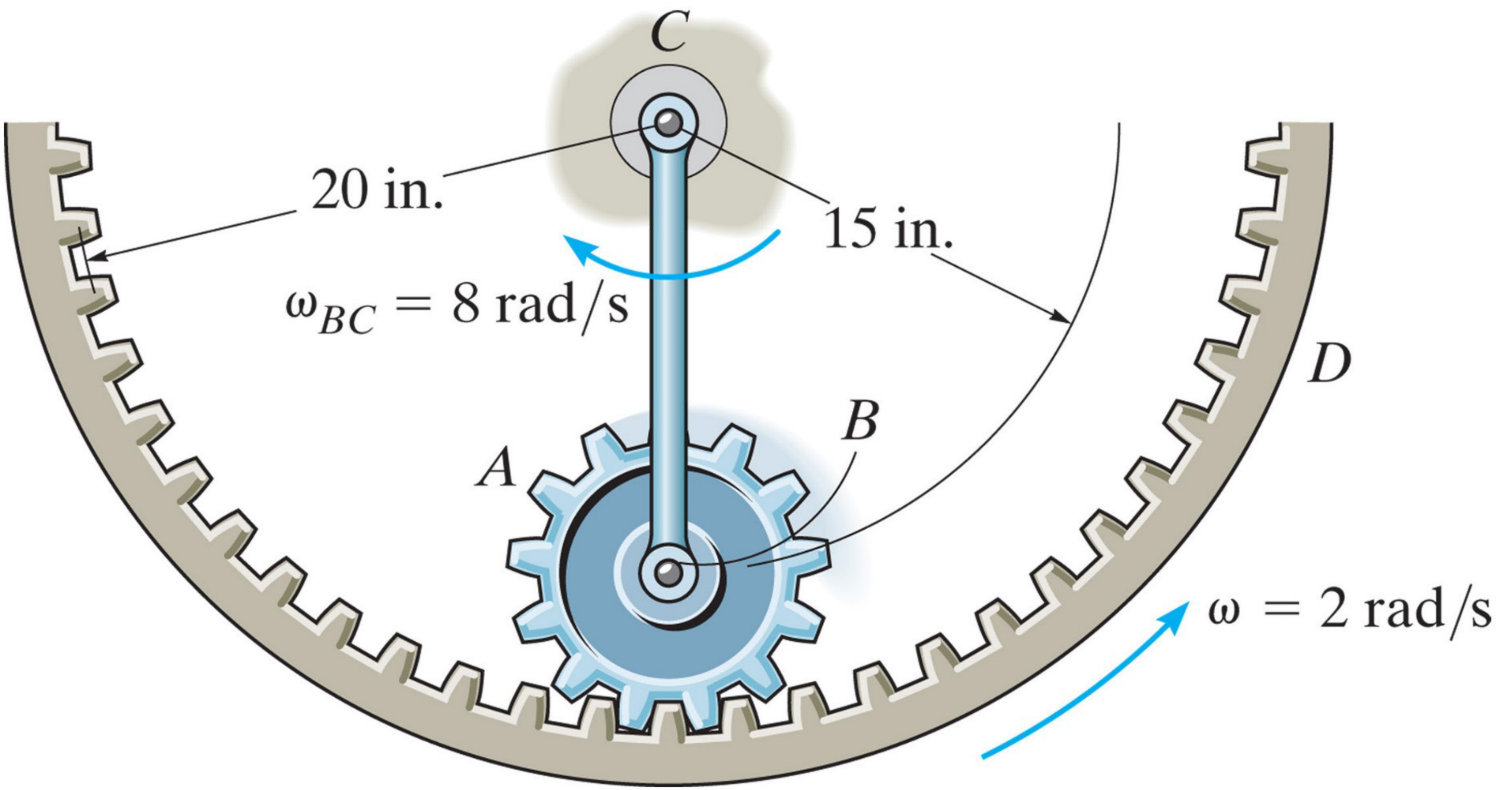
Today's Objectives

- Describe the velocity of a rigid body in terms of translation and rotation components.
- Perform a **relative-motion velocity analysis** of a point on the body.

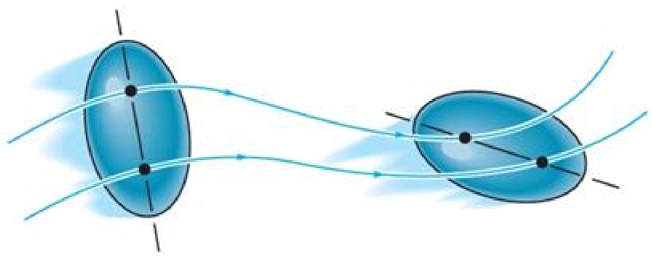
Outline (Pre-Job Brief)

- General Plane Motion
- Translation and Rotation Components of Velocity
- Relative Velocity Analysis
- Examples and Questions
- Summary and Feedback

Relative Velocity Analysis



General Plane Motion

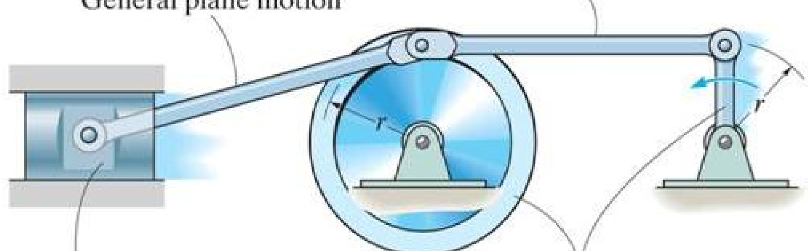


General plane motion

General plane motion: In this case, the body undergoes both translation and rotation.

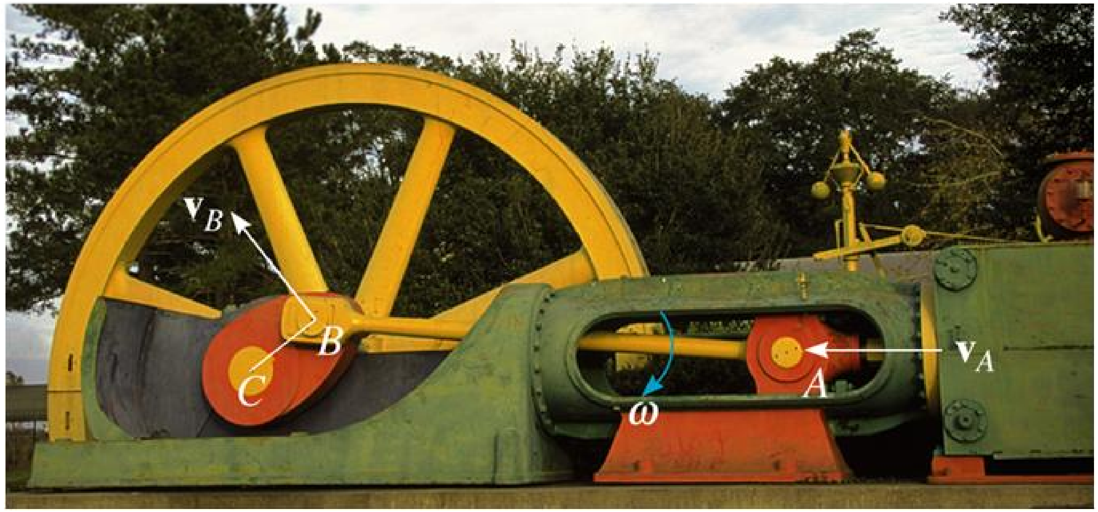
Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.

Motion can be completely specified by knowing both the angular rotation of a line fixed in the body and the motion of a point on the body.



The connecting rod undergoes general plane motion, as it will both translate and rotate.

Applications

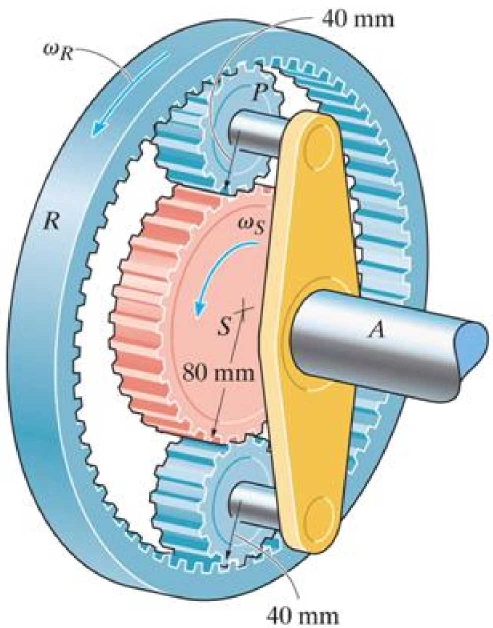


As the slider block A moves horizontally to the left with $\boldsymbol{v}_A$, it causes the link CB to rotate counterclockwise. Thus $\boldsymbol{v}_B$ is directed tangent to its circular path.

Which link is undergoing general plane motion? Link AB or link BC?

How can the angular velocity, ω, of link AB be found?

Applications (continued)

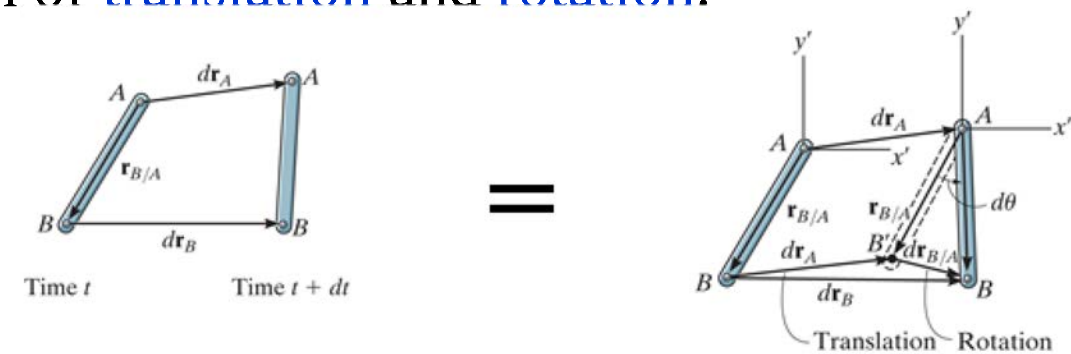


Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds.

How can we relate the angular velocities of the various gears in the system?

Relative Motion Analysis

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.



Point A is called the base point in this analysis. It generally has a known motion. The x'- y' frame translates with the body, but does not rotate. The displacement of point B can be written:

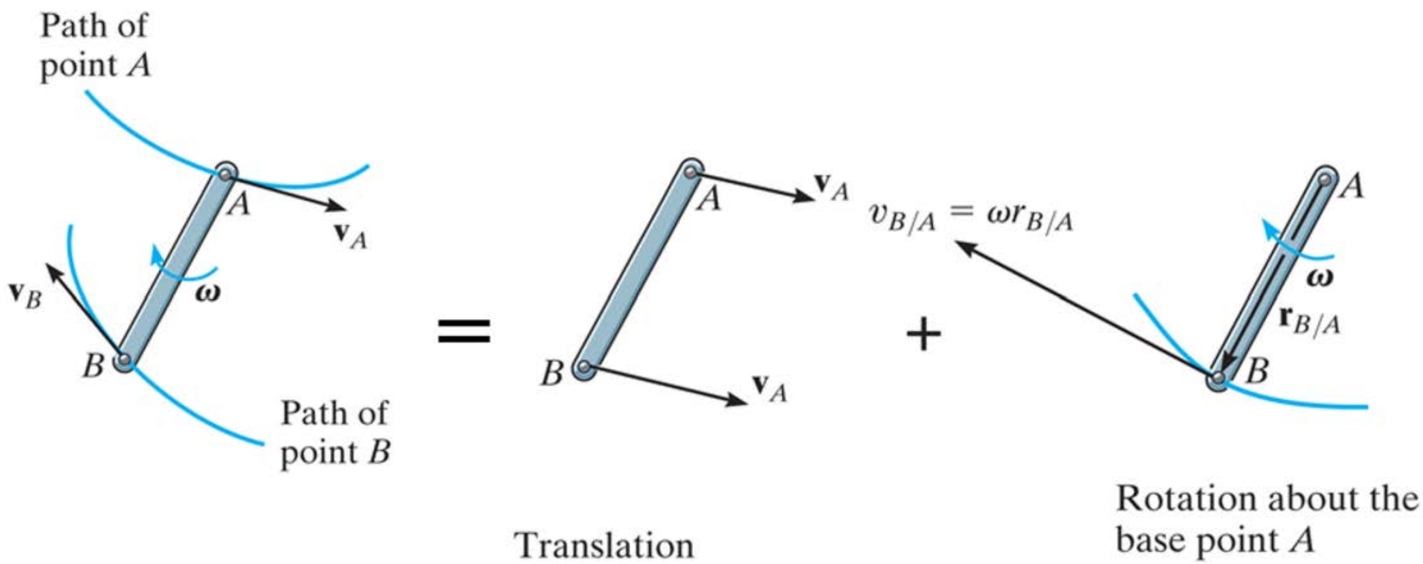
$$d\mathbf{r}_B = d\mathbf{r}_A + d\mathbf{r}_{B/A}$$

Disp. due to translation

Disp. due to translation and rotation

Disp. due to rotation

Relative Motion Analysis: Velocity



The velocity at B is given as : $(d\mathbf{r}_B/dt) = (d\mathbf{r}_A/dt) + (d\mathbf{r}_{B/A}/dt)$ or

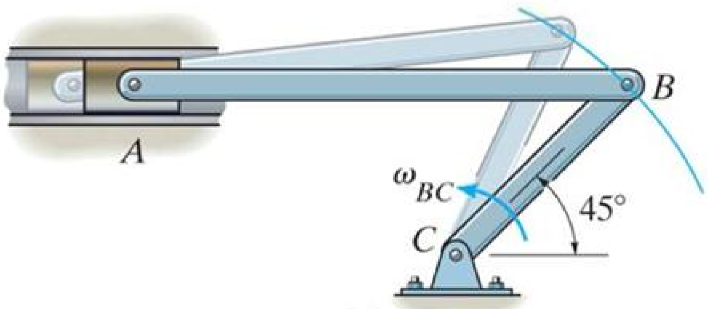
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Since the body is taken as rotating about A,

$$\mathbf{v}_{B/A} = d\mathbf{r}_{B/A}/dt = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

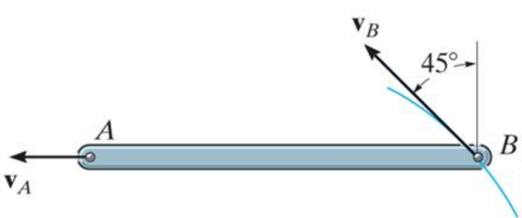
Here $\boldsymbol{\omega}$ will only have a $\mathbf{k}$ component since the axis of rotation is perpendicular to the plane of translation.

Relative Motion Analysis: Velocity



$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

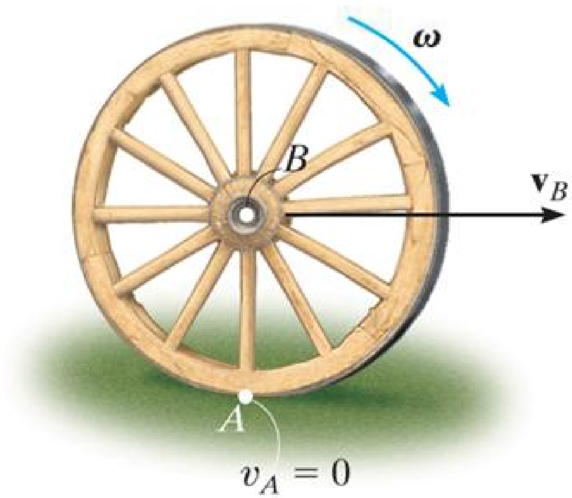
When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.



For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path.

The directions of v_A and v_B are known since they are always tangent to their paths of motion.

Relative Motion Analysis: Velocity



$$\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$$

When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground.

Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $\boldsymbol{v}_B$ has a known direction, e.g., parallel to the surface.

Procedure for Analysis

The relative velocity equation can be applied using scalar x and y component equations or via a Cartesian vector analysis.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector $\boldsymbol{v}_{B/A}$.
2. Write the equation $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$. In the kinematic diagram, represent the vectors graphically by showing their magnitudes and directions underneath each term.
3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.

Procedure for Analysis

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Vector Analysis:

1. Establish the fixed x - y coordinate directions and draw the kinematic diagram of the body, showing the vectors $\boldsymbol{v}_A$, $\boldsymbol{v}_B$, $\boldsymbol{r}_{B/A}$ and $\boldsymbol{\omega}$. If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in Cartesian vector form (CVN) and substitute them into $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{\omega} \times \boldsymbol{r}_{B/A}$. Evaluate the cross product and equate respective $\boldsymbol{i}$ and $\boldsymbol{j}$ components to obtain two scalar equations.

3. If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.

Examples & Questions

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