CEE 271 APPLIED MECHANICS II

Lecture 1: Introduction & Rectilinear Continuous Motion

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Today's Objectives

- Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path

Outline (Pre-Job Brief)

- Brief Review
- Position
- Displacement
- Velocity
- Acceleration
- Examples and Questions
- Summary and Feedback

Review – Problem Solving

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- Clear statement or figure describing problem
- **Identify what is known and what has to be found**
- Create a simple model that represents the original problem. Simplifying assumptions will be needed.
- **Draw a clear figure of the simplified model**
- Use principles of mechanics to create equations that represent the model behavior
- **Estimate your answer**
- Solve these equations using mathematical and computational techniques
- Interpret the results physically and draw conclusions
- **Check the results during and after your solution**
- Present your solution clearly so it is easy to check

Review – Significant Digits

- Digits that carry meaning - round numbers to avoid reporting insignificant figures
- Rules:
  - All non-zero digits are significant
  - Zeros between non-zero digits are significant
  - Leading zeros are never significant
  - Number with decimal point, trailing zeros are significant

Review – Significant Digits

- Rules:
  - For **multiplication and division**, the calculated result should have as many significant digits as the term with the least number of significant digits
  - For **addition and subtraction**, the last significant decimal place of calculated result should match largest last significant decimal place of terms
  - Thumb Rule – only round the final result

Review – Significant Digits

- Round the following to 3 significant digits: 0.025449

A. 0.02

B. 0.03

C. 0.025

D. 0.0254

E. 0.0255

Review – Significant Digits

- Round the following to 3 significant digits: 0.02545

A. 0.0254

B. 0.0255

C. 0.025

D. 0.03

E. None of the above

Review – Units

- US Customary
  - Yard, foot, inch for distance
  - lb, kip for force
  - Slug for mass
  - psi or psf or ksi of ksf for pressure
- SI – Metric
  - Meter, centimeter, millimeter for distance
  - Newton or kN for force
  - Gram or kg for mass
  - Pascal (N/m^2) or MPa (N/mm^2) for pressure

Review – Calculus

- Chain Rule:

$$\dot{y} = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}$$

- If u = u(t) is a function t, then the derivative of u^3 wrt t is:

$$\frac{d}{dt}(u^3) = 3u^2\dot{u}$$

Review – Calculus

- Product Rule:

$$d(uv) = du\, v + u\, dv$$

- If u = u(t), and v = v(t) are functions of t, then:

$$\frac{d}{dt}(uv) = u\dot{v} + \dot{u}v$$

- or

$$\frac{d}{dt}(uv) = \dot{u}v + u\dot{v}$$

Review – Calculus

- Sometimes we need both the Chain Rule and Product Rule. If u = u(t) is a function of t, then the derivative of u^3 wrt t is:

$$\frac{d}{dt}(u^3) = 3u^2\dot{u}$$

- Then the second derivative is:

$$\frac{d}{dt}\left[\frac{d}{dt}(u^3)\right] = (6u\dot{u})\dot{u} + 3u^2\ddot{u}$$

Rectilinear Continuous Motion

(© Lars Johansson/Fotolia)

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Although each of these boats is rather large, from a distance their motion can be analyzed as if each were a particle.

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An Overview of Mechanics

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Dynamics:

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

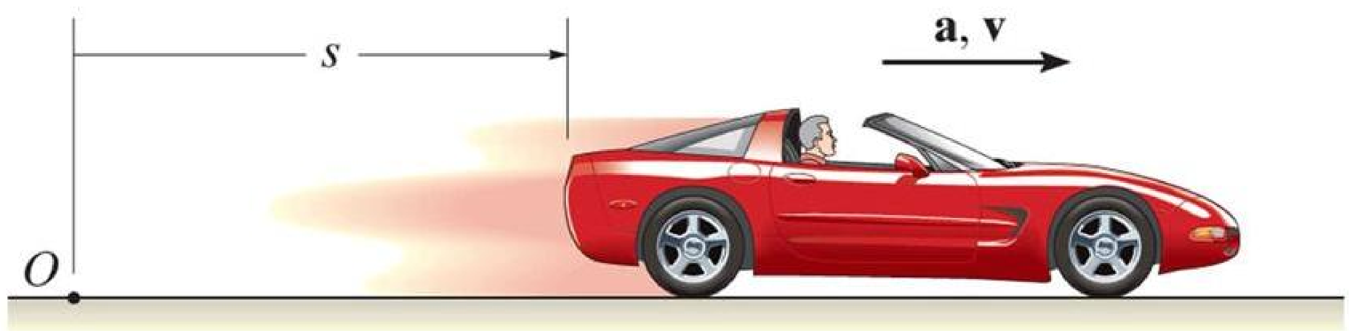
Applications

The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were particles.

Why?

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?

Applications (continued)

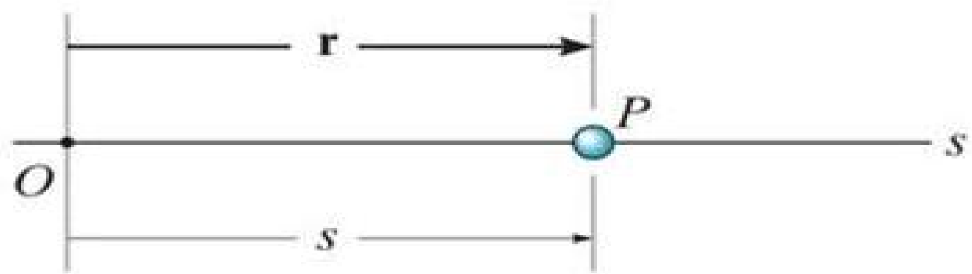


A sports car travels along a straight road. Can we treat the car as a particle?

If the car accelerates at a constant rate, how can we determine its position and velocity at some instant?

Position

A particle travels along a straight-line path defined by the coordinate axis s.

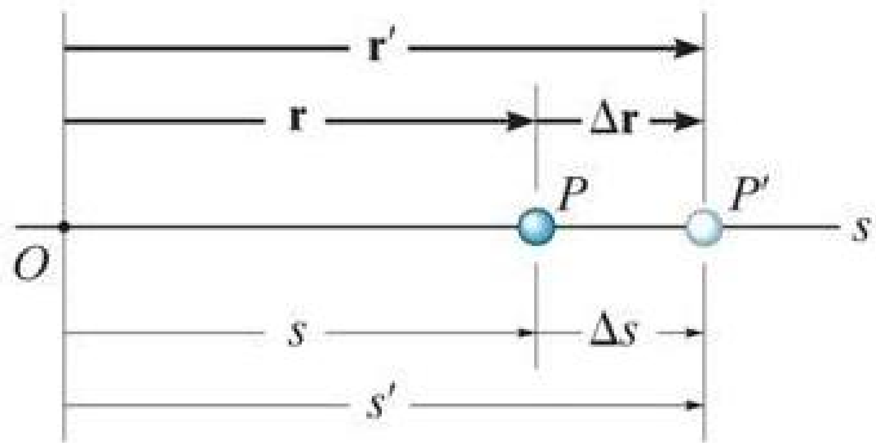


Position

The position of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar s. Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m) or feet (ft).

Displacement

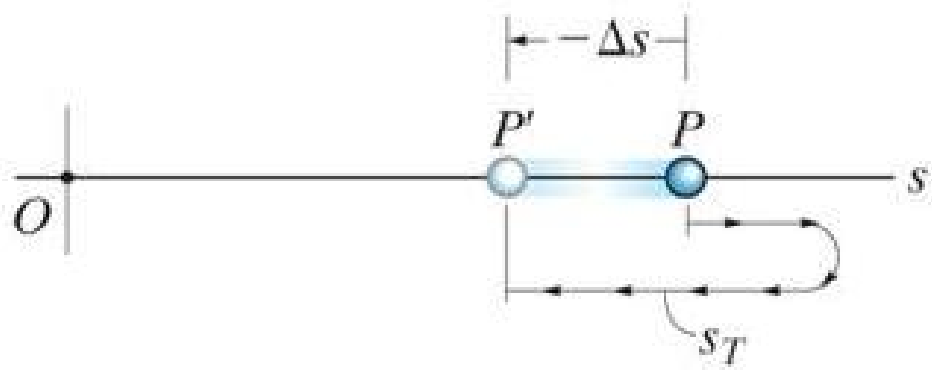
The displacement of the particle is defined as its change in position.



Displacement

Vector form: $\Delta \boldsymbol{r} = \boldsymbol{r}' - \boldsymbol{r}$ Scalar form: $\Delta s = s' - s$

Total Distance Traveled

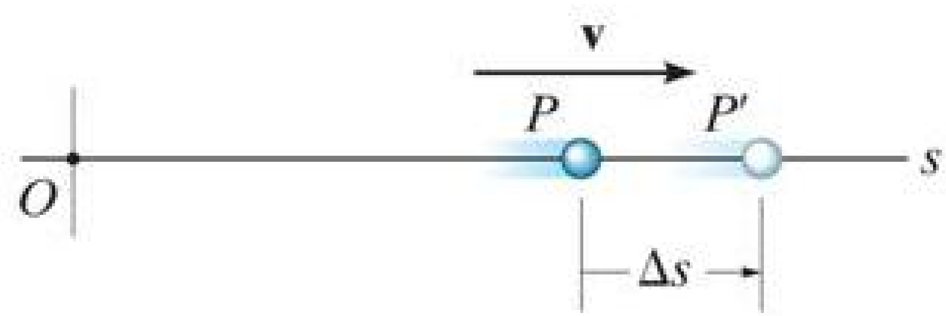


Average velocity and
Average speed

The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.

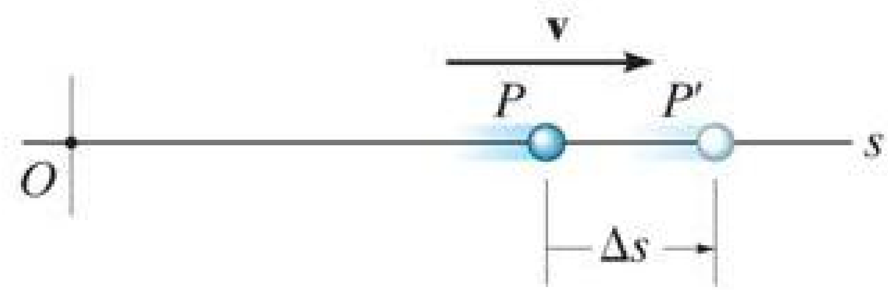
Velocity

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.



Velocity

Velocity



Velocity

The average velocity of a particle during a time interval Δt is

$$\mathbf{v}_{avg} = (\mathbf{r}' - \mathbf{r}) / \Delta t = \Delta \mathbf{r} / \Delta t$$

The instantaneous velocity is the time-derivative of position.

$$\mathbf{v} = d\mathbf{r} / dt$$

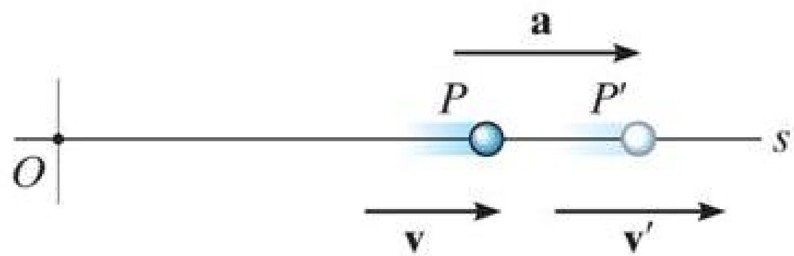
Speed is the magnitude of velocity: $v = ds / dt$

Average speed is the total distance traveled divided by elapsed time:

$$(v_{sp})_{avg} = s_T / \Delta t$$

Acceleration

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s^2 or ft/s^2.



Acceleration

The average acceleration of a particle during a time interval Δt is

$$\boldsymbol{a}_{avg} = (v' - v) / \Delta t = \boldsymbol{\Delta v} / \Delta t$$

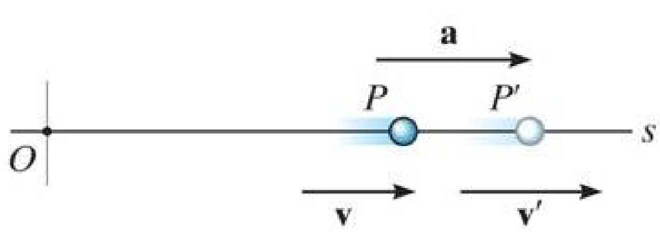
Acceleration

The instantaneous acceleration is the time derivative of velocity.

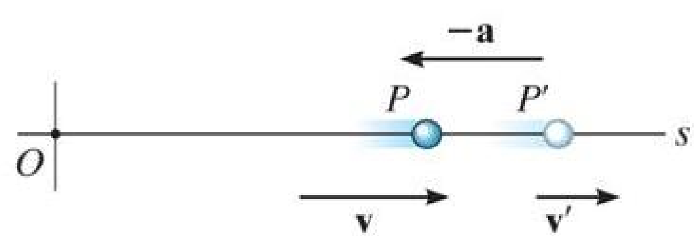
Vector form: $\mathbf{a} = d\mathbf{v} / dt$

Scalar form: $a = dv / dt = d^2s / dt^2$

Acceleration can be positive (speed increasing) or negative (speed decreasing).

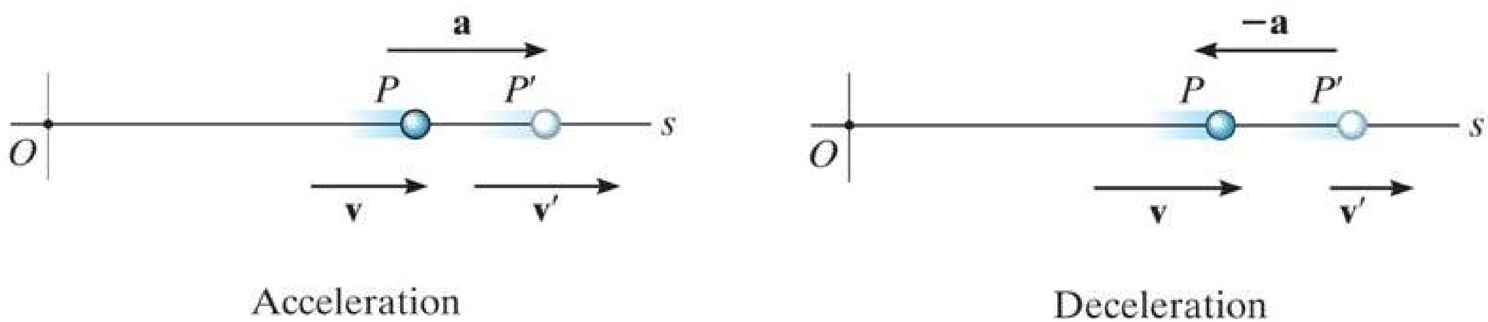


Acceleration



Deceleration

Acceleration



Scalar form: $a = dv / dt = d^2s / dt^2$

As the text indicates, the derivative equations for velocity and acceleration can be manipulated to get

$$a\,ds = v\,dv$$

Summary

Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\, dt \quad \text{or} \quad \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt$$

Note that s_o and v_o represent the initial position and velocity of the particle at $t = 0$.

Constant Acceleration

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81\ m/s^2 = 32.2\ ft/s^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c\, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2)\, a_c\, t^2$$

$$\int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a_c\, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

Examples & Questions

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