CEE 271 APPLIED MECHANICS II

Lecture 19: Translation & Rotation about a Fixed Axis

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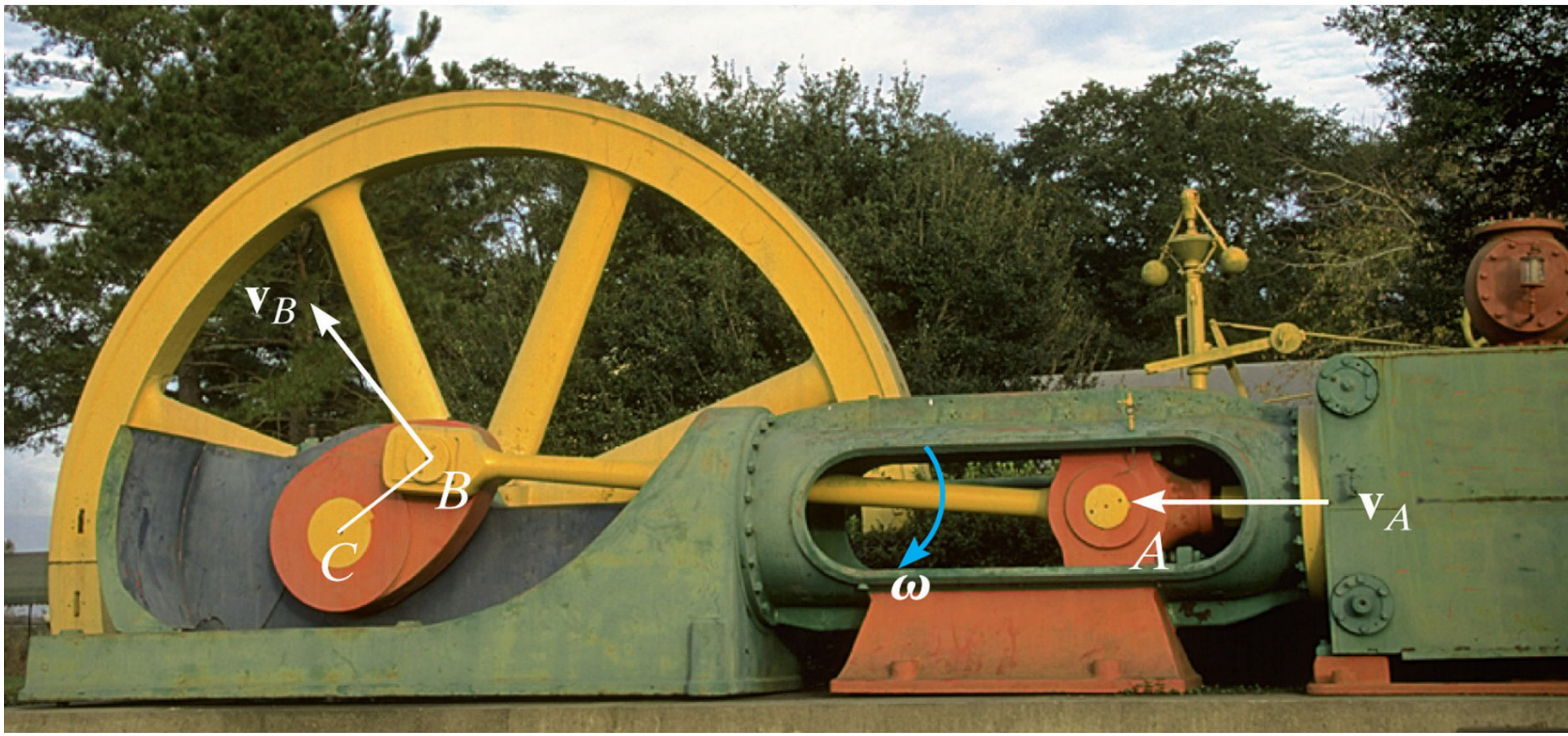
Today's Objectives

- Analyze the kinematics of a rigid body undergoing planar translation or rotation about a fixed axis.

Outline (Pre-Job Brief)

- Types of Rigid-Body Motion
- Planar Translation
- Rotation about a Fixed Axis
- Examples and Questions
- Summary and Feedback

Translation & Rotation about a Fixed Axis



Applications

Passengers on this amusement ride are subjected to curvilinear translation since the vehicle moves in a circular path but they always remains upright.

If the angular motion of the rotating arms is known, how can we determine the velocity and acceleration experienced by the passengers? Why would we want to know these values?

Does each passenger feel the same acceleration?

Applications (continued)

Gears, pulleys and cams, which rotate about fixed axes, are often used in machinery to generate motion and transmit forces. The angular motion of these components must be understood to properly design the system.

To do this design, we need to relate the angular motions of contacting bodies that rotate about different fixed axes. How is this different than the analyses we did in earlier chapters?

Rigid Body Motion

There are cases where an object cannot be treated as a particle. In these cases the size or shape of the body must be considered. Rotation of the body about its center of mass requires a different approach.

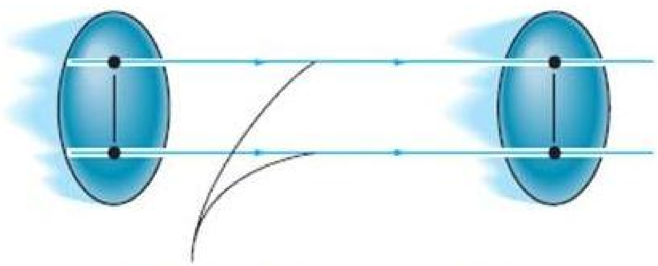
For example, in the design of gears, cams, and links in machinery or mechanisms, rotation of the body is an important aspect in the analysis of motion.

We will now start to study rigid body motion. The analysis will be limited to planar motion.

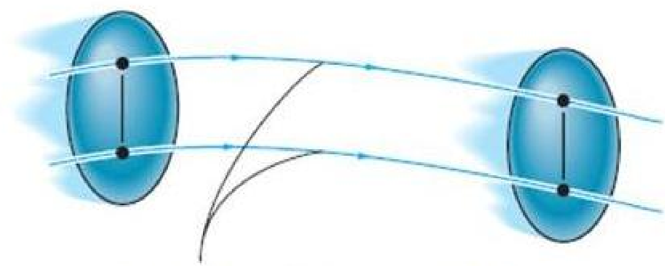
A body is said to undergo planar motion when all parts of the body move along paths equidistant from a fixed plane.

Planar Rigid Body Motion

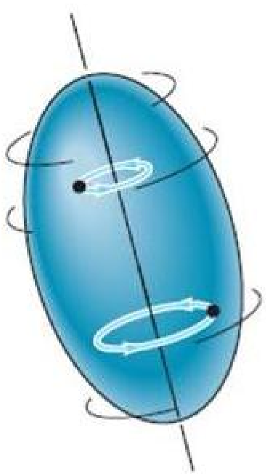
There are three types of planar rigid body motion.



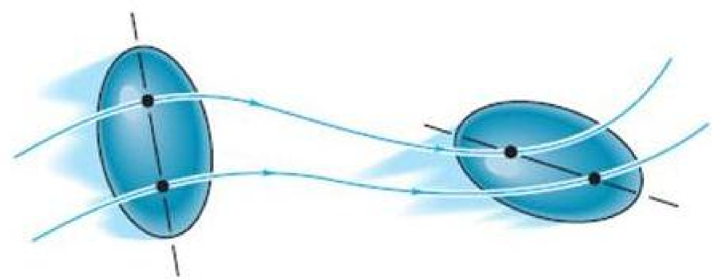
Path of rectilinear translation



Path of curvilinear translation

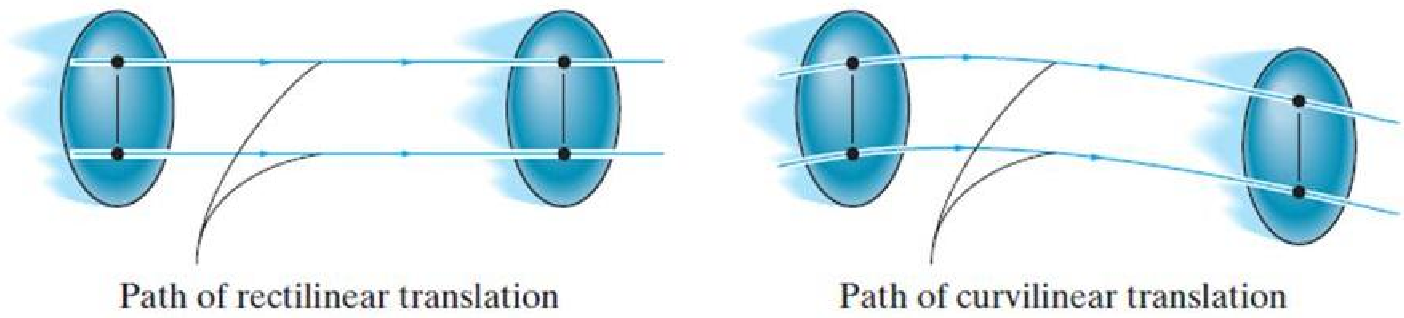


Rotation about a fixed axis



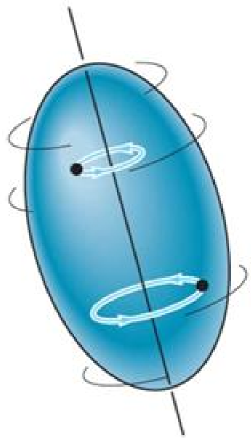
General plane motion

Planar Rigid Body Motion



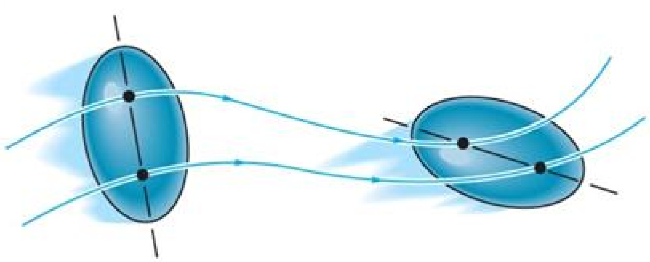
Translation: Translation occurs if every line segment on the body remains parallel to its original direction during the motion. When all points move along straight lines, the motion is called rectilinear translation. When the paths of motion are curved lines, the motion is called curvilinear translation.

Planar Rigid Body Motion



Rotation about a fixed axis

Rotation about a fixed axis: In this case, all the particles of the body, except those on the axis of rotation, move along circular paths in planes perpendicular to the axis of rotation.

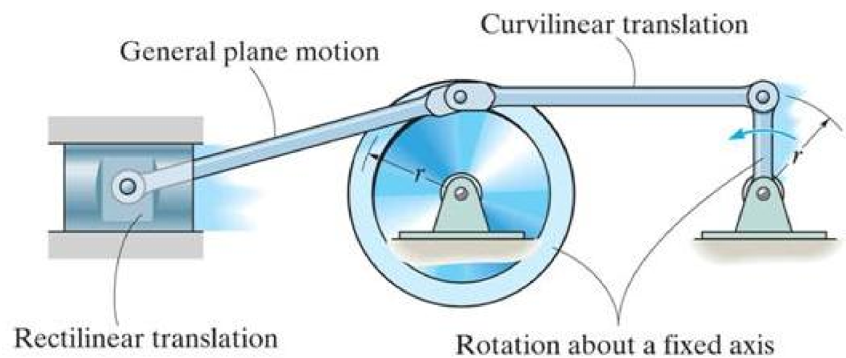


General plane motion

General plane motion: In this case, the body undergoes both translation and rotation. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.

Planar Rigid Body Motion

An example of bodies undergoing the three types of motion is shown in this mechanism.



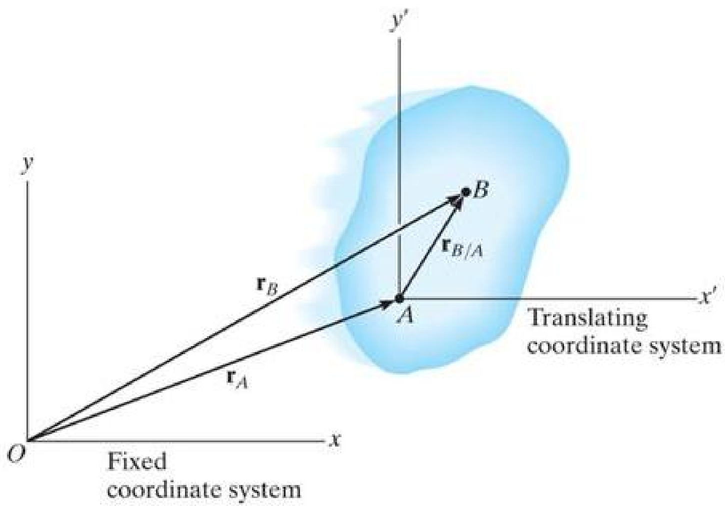
The wheel and crank undergo rotation about a fixed axis. In this case, both axes of rotation are at the location of the pins and perpendicular to the plane of the figure.

The piston undergoes rectilinear translation since it is constrained to slide in a straight line.

The connecting rod undergoes curvilinear translation, since it will remain horizontal as it moves along a circular path.

The connecting rod undergoes general plane motion, as it will both translate and rotate.

Translation



The positions of two points A and B on a translating body can be related by

$$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$$

where $\boldsymbol{r}_A$ & $\boldsymbol{r}_B$ are the absolute position vectors defined from the fixed x-y coordinate system, and $\boldsymbol{r}_{B/A}$ is the relative-position vector between B and A.

The velocity at B is $\boldsymbol{v}_B = \boldsymbol{v}_A + d\boldsymbol{r}_{B/A}/dt$.

Now $d\boldsymbol{r}_{B/A}/dt = 0$ since $\boldsymbol{r}_{B/A}$ is constant. So, $\boldsymbol{v}_B = \boldsymbol{v}_A$, and by following similar logic, $\mathbf{a}_B = \mathbf{a}_A$.

Note, all points in a rigid body subjected to translation move with the same velocity and acceleration.

Rotation: Fixed Axis

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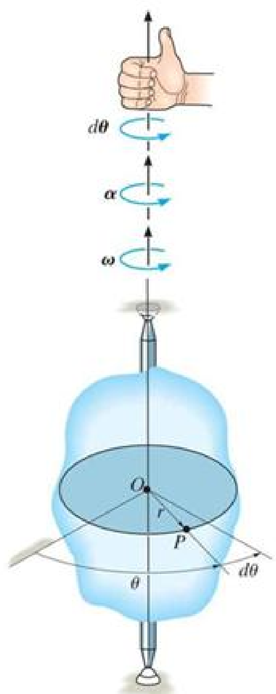


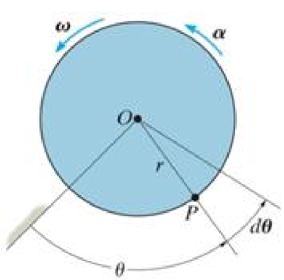
When a body rotates about a fixed axis, any point P in the body travels along a circular path. The angular position of P is defined by θ.

The change in angular position, $d\theta$, is called the angular displacement, with units of either radians or revolutions. They are related by

$$1 \text{ revolution} = (2\pi) \text{ radians}$$

Angular velocity, ω, is obtained by taking the time derivative of angular displacement:

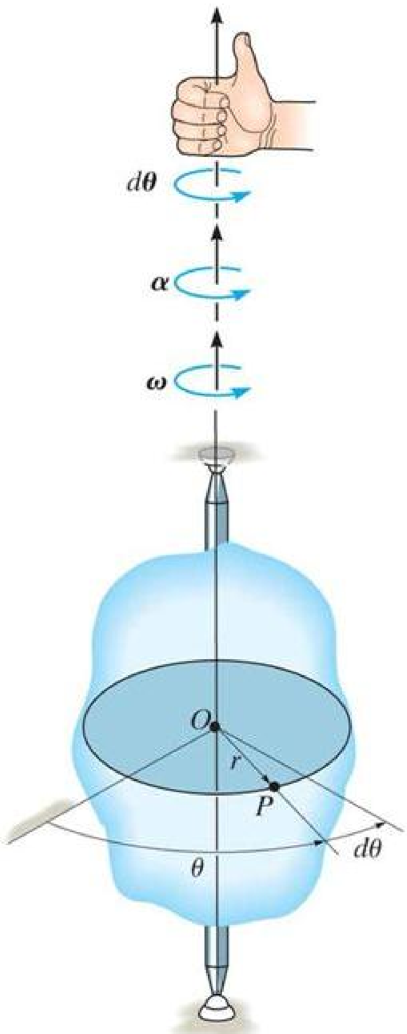
$\omega = d\theta/dt$ (rad/s) +



Similarly, angular acceleration is

$\alpha = d^2\theta/dt^2 = d\omega/dt$ or $\alpha = \omega(d\omega/d\theta)$ rad/s^2 +

Rotation: Fixed Axis

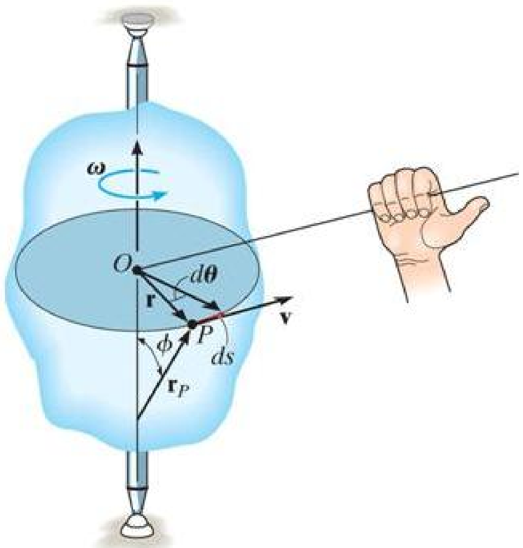


If the angular acceleration of the body is constant, $\alpha = \alpha_C$, the equations for angular velocity and acceleration can be integrated to yield the set of algebraic equations below.

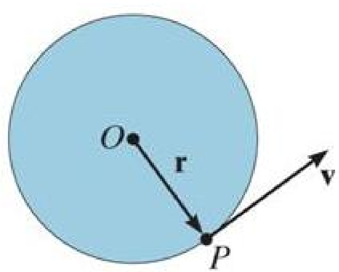
$$\omega = \omega_0 + \alpha_C t$$
$$\theta = \theta_0 + \omega_0 t + 0.5\, \alpha_C t^2$$
$$\omega^2 = (\omega_0)^2 + 2\alpha_C (\theta - \theta_0)$$

θ_0 and ω_0 are the initial values of the body's angular position and angular velocity. Note these equations are very similar to the constant acceleration relations developed for the rectilinear motion of a particle.

Rotation: Fixed Axis



The magnitude of the velocity of P is equal to ωr (the text provides the derivation). The velocity's direction is tangent to the circular path of P.

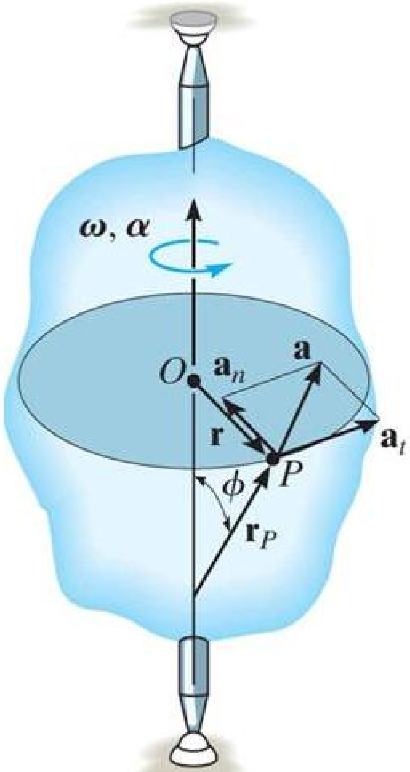


In the vector formulation, the magnitude and direction of ***v*** can be determined from the cross product of $\boldsymbol{\omega}$ and $\boldsymbol{r}_p$. Here $\boldsymbol{r}_p$ is a vector from any point on the axis of rotation to P.

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_p = \boldsymbol{\omega} \times \boldsymbol{r}$$

The direction of ***v*** is determined by the right-hand rule.

Rotation: Fixed Axis

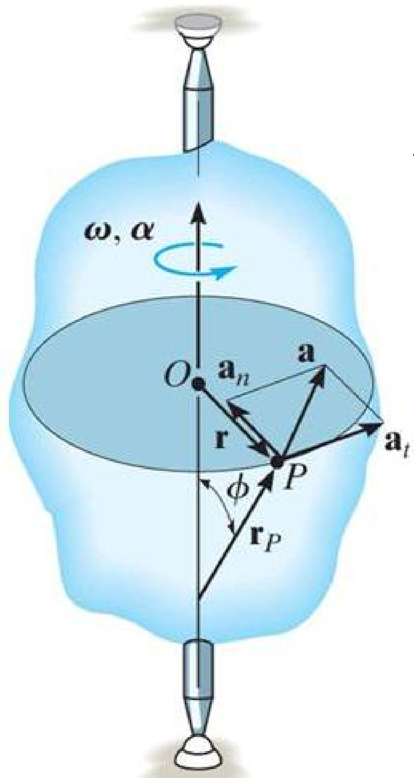


The acceleration of P is expressed in terms of its normal (**a**$_n$) and tangential (**a**$_t$) components. In scalar form, these are $a_t = \alpha\ r$ and $a_n = \omega^2\ r$.

The tangential component, **a**$_t$, represents the time rate of change in the velocity's magnitude. It is directed tangent to the path of motion.

The normal component, **a**$_n$, represents the time rate of change in the velocity's direction. It is directed toward the center of the circular path.

Rotation: Fixed Axis



Using the vector formulation, the acceleration of P can also be defined by differentiating the velocity.

$$\mathbf{a} = d\boldsymbol{v}/dt = d\boldsymbol{\omega}/dt \times \boldsymbol{r}_P + \boldsymbol{\omega} \times d\boldsymbol{r}_P/dt$$

$$= \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P)$$

It can be shown that this equation reduces to

$$\mathbf{a} = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2\boldsymbol{r} = \mathbf{a}_t + \mathbf{a}_n$$

The magnitude of the acceleration vector is $a = \sqrt{(a_t)^2 + (a_n)^2}$

Procedure

Establish a sign convention along the axis of rotation.

If a relationship is known between any two of the variables (α, ω, θ, or t), the other variables can be determined from the equations: $\omega = d\theta/dt$ $\quad \alpha = d\omega/dt$ $\quad \alpha\, d\theta = \omega\, d\omega$

If α is constant, use the equations for constant angular acceleration.

To determine the motion of a point, the scalar equations $v = \omega r$, $a_t = \alpha r$, $a_n = \omega^2 r$, and $a = \sqrt{(a_t)^2 + (a_n)^2}$ can be used.

Alternatively, the vector form of the equations can be used (with $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ components).

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}_P = \boldsymbol{\omega} \times \boldsymbol{r}$$

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n = \boldsymbol{\alpha} \times \boldsymbol{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}_P) = \boldsymbol{\alpha} \times \boldsymbol{r} - \omega^2 \boldsymbol{r}$$

Examples & Questions

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