CEE 271 APPLIED MECHANICS II

Lecture 18: Angular Impulse & Momentum

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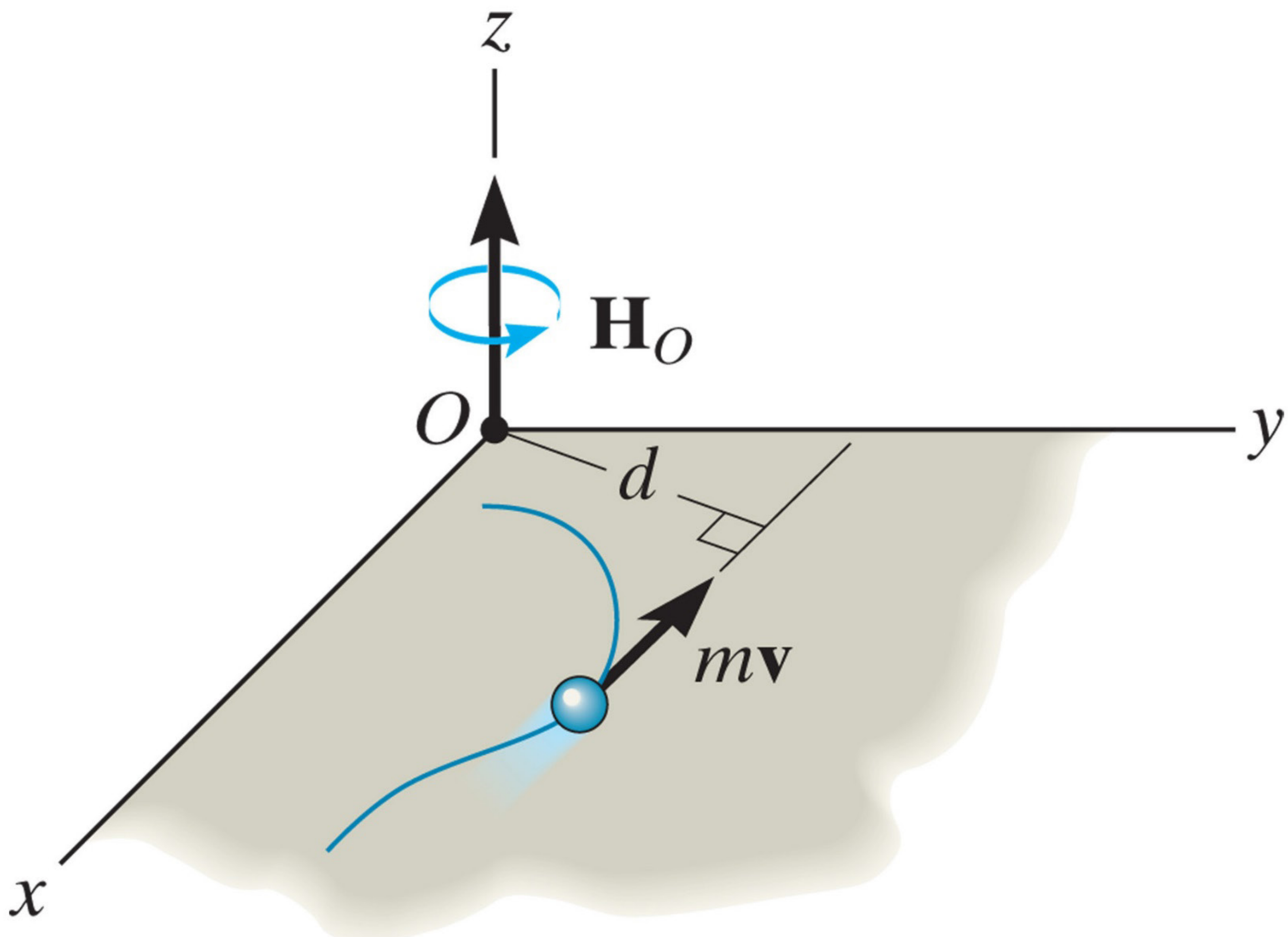
Today's Objectives

- Determine the angular momentum of a particle and apply the principle of angular impulse & momentum.
- Use conservation of angular momentum to solve problems.

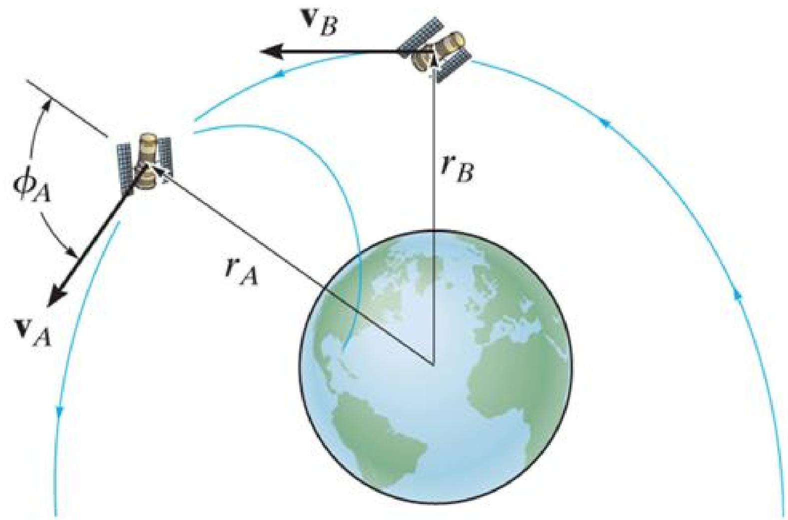
Outline (Pre-Job Brief)

- Angular Momentum
- Angular Impulse and Momentum Principle
- Conservation of Angular Momentum
- Examples and Questions
- Summary and Feedback

Angular Impulse & Momentum



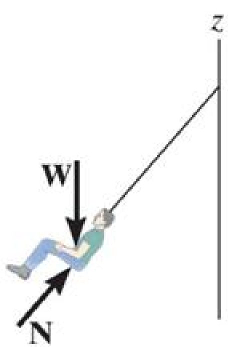
Applications



Planets and most satellites move in elliptical orbits. This motion is caused by gravitational attraction forces. Since these forces act in pairs, the sum of the moments of the forces acting on the system will be zero. This means that angular momentum is conserved.

If the angular momentum is constant, does it mean the linear momentum is also constant? Why or why not?

Applications (continued)

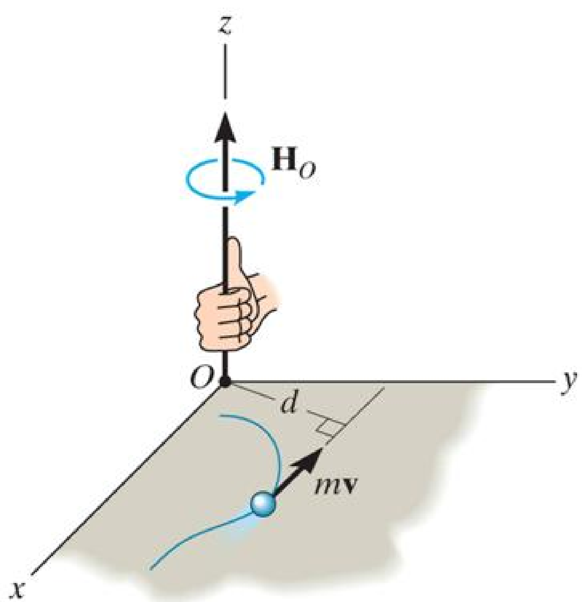


The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z-axis). As shown on the free body diagram, the line of action of the normal force, N, passes through the z-axis and the weight's line of action is parallel to it. Therefore, the sum of moments of these two forces about the z-axis is zero.

If the passenger moves away from the z-axis, will his speed increase or decrease? Why?

Angular Momentum

The angular momentum of a particle about point O is defined as the “moment” of the particle’s linear momentum about O.



$$\boldsymbol{H}_o = \boldsymbol{r} \times m\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of $\boldsymbol{H}_o$ is $(H_o)_z = mv\, d$

Moment & Angular Momentum

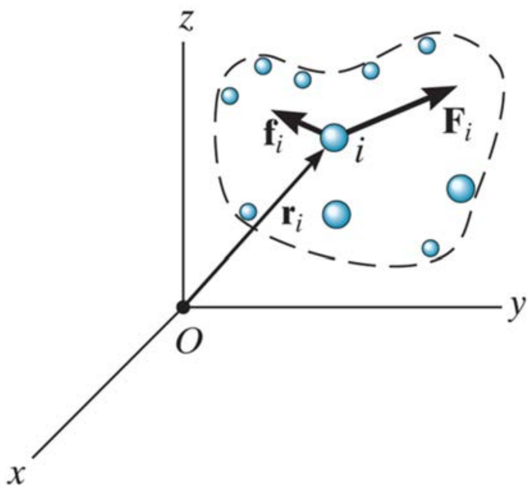
The resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum. Showing the time derivative using the familiar "dot" notation results in the equation

$$\sum \boldsymbol{F} = \dot{\boldsymbol{L}} = m\dot{\boldsymbol{v}}$$

We can prove that the resultant moment acting on the particle about point O is equal to the time rate of change of the particle's angular momentum about point O or

$$\sum \boldsymbol{M}_o = \boldsymbol{r} \times \sum \boldsymbol{F} = \dot{\boldsymbol{H}}_o$$

System of Particles



$$\sum \boldsymbol{M}_o = r \times \boldsymbol{F} = \dot{\boldsymbol{H}}_o$$

The same form of the equation can be derived for the system of particles.

The forces acting on the i-th particle of the system consist of a resultant external force $\boldsymbol{F}_i$ and a resultant internal force $\boldsymbol{f}_i$.

Then, the moments of these forces for the particles can be written as $\sum(r_i \times \boldsymbol{F}_i) + \sum(r_i \times \boldsymbol{f}_i) = \sum(\dot{\boldsymbol{H}}_i)_o$

The second term is zero since the internal forces occur in equal but opposite collinear pairs. Thus,

$$\sum \boldsymbol{M}_o = \sum(r_i \times \boldsymbol{F}_i) = \sum(\dot{\boldsymbol{H}}_i)_o$$

Principle of Angular Impulse and Momentum

Considering the relationship between moment and time rate of change of angular momentum

$$\sum \boldsymbol{M}_o = \dot{\boldsymbol{H}}_o = d\boldsymbol{H}_o/dt$$

By integrating between the time interval t_1 to t_2

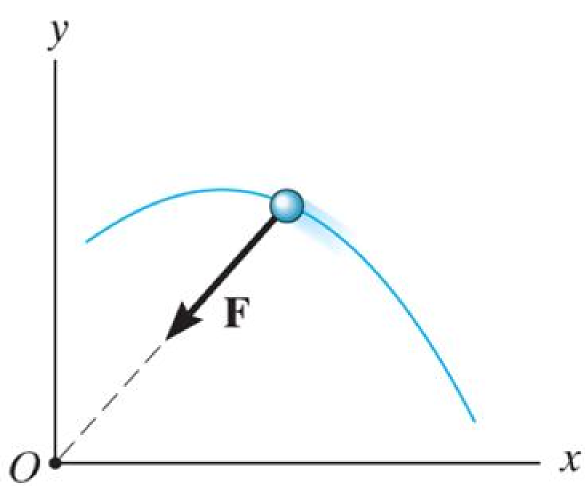
$$\sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2 - (\boldsymbol{H}_o)_1 \quad \text{or} \quad (\boldsymbol{H}_o)_1 + \sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2$$

This equation is referred to as the principle of angular impulse and momentum. The second term on the left side, $\sum\int M_o \, dt$, is called the angular impulse. In cases of 2D motion, it can be applied as a scalar equation using components about the z-axis.

Conservation of Angular Momentum

When the sum of angular impulses acting on a particle or a system of particles is zero during the time t_1 to t_2, the angular momentum is conserved. Thus,

$$(\boldsymbol{H}_O)_1 = (\boldsymbol{H}_O)_2$$



An example of this condition occurs when a particle is subjected only to a central force. In the figure, the force ***F*** is always directed toward point O. Thus, the angular impulse of ***F*** about O is always zero, and angular momentum of the particle about O is conserved.

Examples & Questions

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