CEE 271 APPLIED MECHANICS II

Lecture 17: Impact

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Today's Objectives

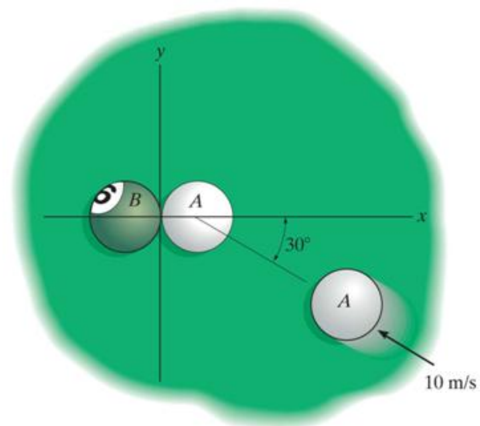
- Understand and analyze the mechanics of impact.
- Analyze the motion of bodies undergoing a collision, in both central and oblique cases of impact.

Outline (Pre-Job Brief)

- Central Impact
- Coefficient of Restitution
- Oblique Impact
- Examples and Questions
- Summary and Feedback

Impact

Applications

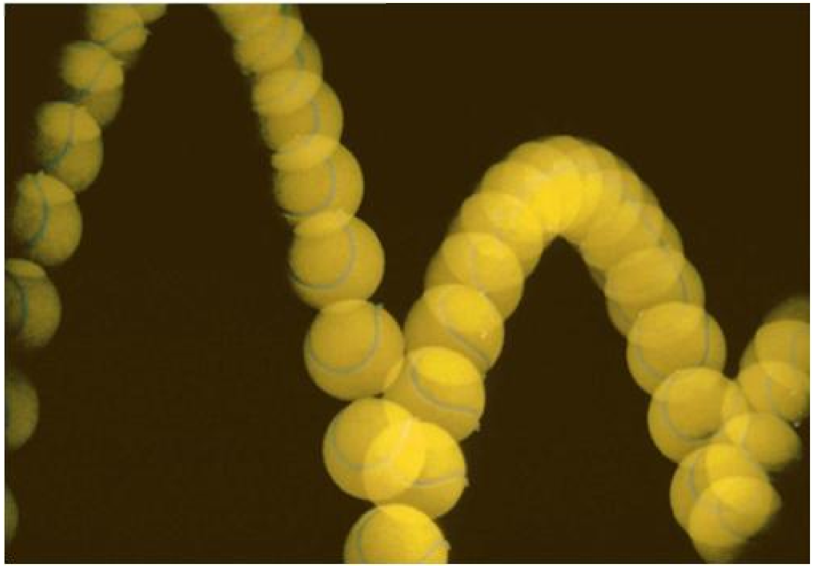


In the game of billiards, it is important to be able to predict the trajectory and speed of a ball after it is struck by another ball.

If we know the velocity of ball A before the impact, how can we determine the magnitude and direction of the velocity of ball B after the impact?

What parameters would we need to know to do this?

Applications

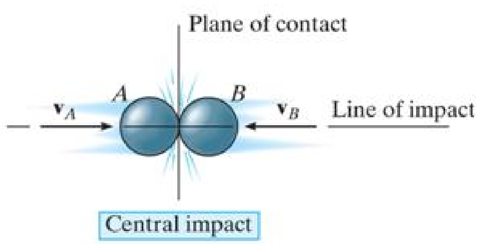


The quality of a tennis ball is measured by the height of its bounce. This can be quantified by the coefficient of restitution of the ball.

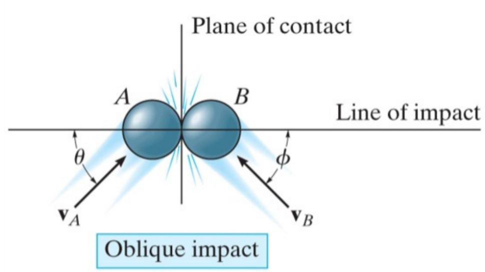
If the height from which the ball is dropped and the height of its resulting bounce are known, how can we determine the coefficient of restitution of the ball?

Impact

Impact occurs when two bodies collide during a very short time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The line of impact is a line through the mass centers of the colliding particles. In general, there are two types of impact:



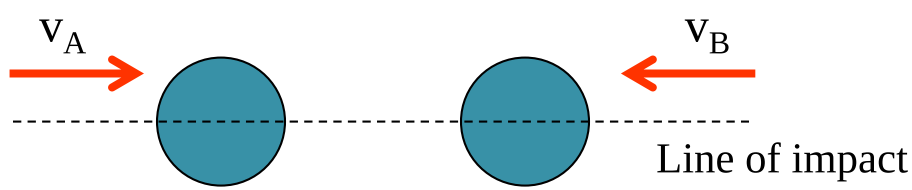
Central impact occurs when the directions of motion of the two colliding particles are along the line of impact.

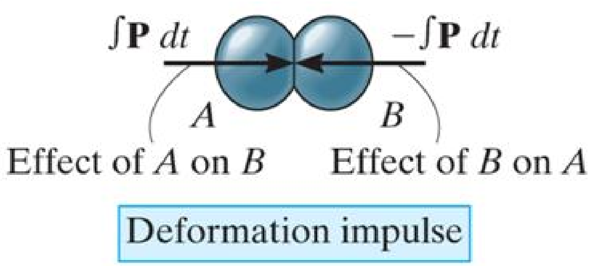


Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.

Central Impact

Central impact happens when the velocities of the two objects are along the line of impact (recall that the line of impact is a line through the particles' mass centers).





Once the particles contact, they may deform if they are non-rigid. In any case, energy is transferred between the two particles.

There are two primary equations used when solving impact problems. The textbook provides extensive detail on their derivation.

Central Impact (continued)

In most problems, the initial velocities of the particles, $(v_A)_1$ and $(v_B)_1$, are known, and it is necessary to determine the final velocities, $(v_A)_2$ and $(v_B)_2$. So the first equation used is the conservation of linear momentum, applied along the line of impact.

$$(m_A\, v_A)_1 + (m_B\, v_B)_1 = (m_A\, v_A)_2 + (m_B\, v_B)_2$$

This provides one equation, but there are usually two unknowns, $(v_A)_2$ and $(v_B)_2$. So another equation is needed. The principle of impulse and momentum is used to develop this equation, which involves the coefficient of restitution, or e.

Central Impact (continued)

The coefficient of restitution, e, is the ratio of the particles' relative separation velocity after impact, $(v_B)_2 – (v_A)_2$, to the particles' relative approach velocity before impact, $(v_A)_1 – (v_B)_1$. The coefficient of restitution is also an indicator of the energy lost during the impact.

The equation defining the coefficient of restitution, *e*, is

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

If a value for *e* is specified, this relation provides the second equation necessary to solve for $(v_A)_2$ and $(v_B)_2$.

Coefficient of Restitution

In general, e has a value between zero and one. The two limiting conditions can be considered:

Elastic impact ($e = 1$): In a perfectly elastic collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition cannot be achieved.

Plastic impact ($e = 0$): In a plastic impact, the relative separation velocity is zero. The particles stick together and move with a common velocity after the impact.

Some typical values of e are:

Steel on steel: 0.5 – 0.8

Wood on wood: 0.4 – 0.6

Lead on lead: 0.12 – 0.18

Glass on glass: 0.93 – 0.95

Impact: Energy Losses

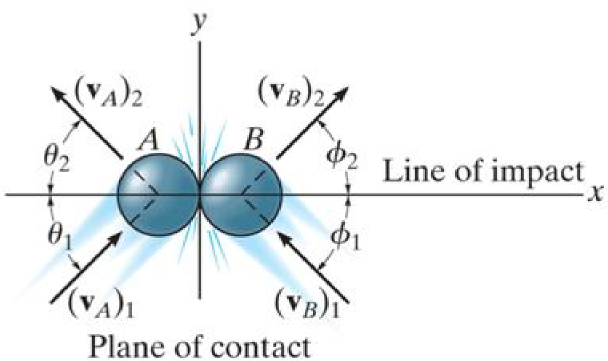
Once the particles' velocities before and after the collision have been determined, the energy loss during the collision can be calculated on the basis of the difference in the particles' kinetic energy. The energy loss is

$$\sum U_{1\text{-}2} = \sum T_2 - \sum T_1 \quad \text{where } T_i = 0.5 m_i (v_i)^2$$

During a collision, some of the particles' initial kinetic energy will be lost in the form of heat, sound, or due to localized deformation.

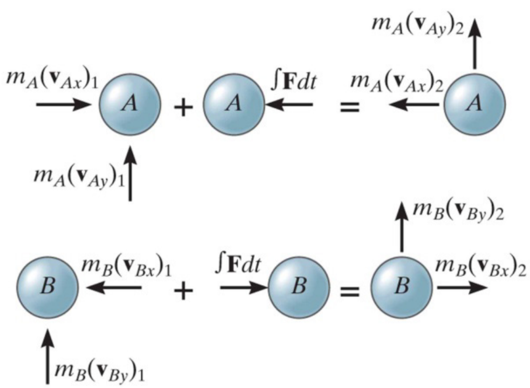
In a plastic collision ($e = 0$), the energy lost is a maximum, although it does not necessarily go to zero. Why?

Oblique Impact



In an oblique impact, one or both of the particles' motion is at an angle to the line of impact. Typically, there will be four unknowns: the magnitudes and directions of the final velocities.

The four equations required to solve for the unknowns are:



Conservation of momentum and the coefficient of restitution equation are applied along the line of impact (x-axis):

$$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$$

Momentum of each particle is conserved in the direction perpendicular to the line of impact (y-axis):

$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \text{and} \quad m_B(v_{By})_1 = m_B(v_{By})_2$$

Procedure for Analysis

In most impact problems, the initial velocities of the particles and the coefficient of restitution, *e*, are known, with the final velocities to be determined.

Define the x-y axes. Typically, the x-axis is defined along the line of impact and the y-axis is in the plane of contact perpendicular to the x-axis.

For both central and oblique impact problems, the following equations apply along the line of impact (x-dir.):

$$\sum m(v_x)_1 = \sum m(v_x)_2 \quad \text{and} \quad e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$$

For oblique impact problems, the following equations are also required, applied perpendicular to the line of impact (y-dir.):

$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \text{and} \quad m_B(v_{By})_1 = m_B(v_{By})_2$$

Examples & Questions

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