CEE 271 APPLIED MECHANICS II

Lecture 15: Linear Impulse & Momentum

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Today's Objectives

- Calculate the linear momentum of a particle and linear impulse of a force.
- Apply the principle of linear impulse and momentum.

Outline (Pre-Job Brief)

- Linear Momentum and Impulse
- Principle of Linear Impulse and Momentum
- Examples and Questions
- Summary and Feedback

Linear Impulse & Momentum

Applications

A dent in an trailer fender can be removed using an impulse tool, which delivers a force over a very short time interval. To do so, the weight is gripped and jerked upwards, striking the stop ring.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion?

Sure!

Applications (continued)

A good example of impulse is the action of hitting a ball with a bat.

The impulse is the average force exerted by the bat multiplied by the time the bat and ball are in contact.

Is the impulse a vector? Is the impulse pointing in the same direction as the force being applied?

Given the situation of hitting a ball, how can we predict the resultant motion of the ball?

Applications (continued)

When a stake is struck by a sledgehammer, a large impulse force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

Principle of Linear Impulse and Momentum

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The next method we will consider for solving particle kinetics problems is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).

Principle of Linear Impulse and Momentum

The principle of linear impulse and momentum is obtained by integrating the equation of motion with respect to time. The equation of motion can be written

$$\sum \boldsymbol{F} = m\,\mathbf{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

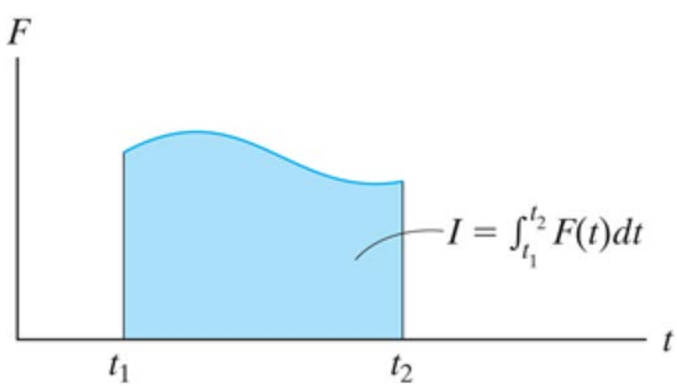
$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

This equation represents the principle of linear impulse and momentum. It relates the particle's final velocity ($\boldsymbol{v}_2$) and initial velocity ($\boldsymbol{v}_1$) and the forces acting on the particle as a function of time.

Principle of Linear Impulse and Momentum

Linear momentum: The vector m***v*** is called the linear momentum, denoted as ***L***. This vector has the same direction as ***v***. The linear momentum vector has units of (kg·m)/s or (slug·ft)/s.

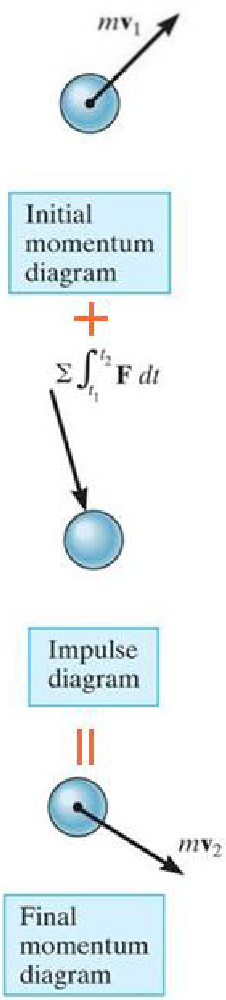
Linear impulse: The integral ∫***F*** dt is the linear impulse, denoted ***I***. It is a vector quantity measuring the effect of a force during its time interval of action. ***I*** acts in the same direction as ***F*** and has units of N·s or lb·s.



The impulse may be determined by direct integration. Graphically, it can be represented by the area under the force versus time curve. If ***F*** is constant, then

$$\boldsymbol{I} = \boldsymbol{F}\,(t_2 - t_1)\ .$$

Principle of Linear Impulse and Momentum



The principle of linear impulse and momentum in vector form is written as

$$m\boldsymbol{v}_1 + \sum\int_{t_1}^{t_2} \boldsymbol{F}\,dt = m\boldsymbol{v}_2$$

The particle's initial momentum plus the sum of all the impulses applied from t_1 to t_2 is equal to the particle's final momentum.

The two momentum diagrams indicate direction and magnitude of the particle's initial and final momentum, $m\boldsymbol{v}_1$ and $m\boldsymbol{v}_2$. The impulse diagram is similar to a free body diagram, but includes the time duration of the forces acting on the particle.

Scalar Equations

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

Problem Solving

Establish the x, y, z coordinate system.

Draw the particle's free body diagram and establish the direction of the particle's initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.

Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.

Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

Examples & Questions

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