CEE 271 APPLIED MECHANICS II

Lecture 14: Conservation of Energy

Department of Civil & Environmental Engineering
University of Hawaiʻi at Mānoa

Today's Objectives

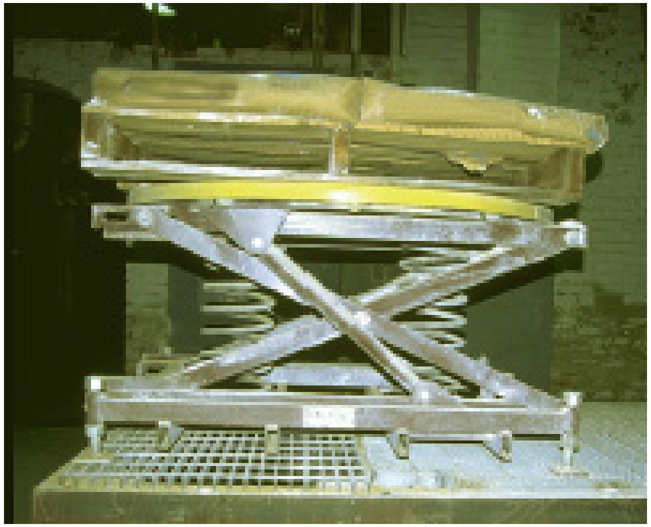
- Use the concept of conservative forces and determine the potential energy of such forces.
- Apply the principle of conservation of energy.

Outline (Pre-Job Brief)

- Conservative Force
- Potential Energy
- Conservation of Energy
- Examples and Questions
- Summary and Feedback

Conservation of Energy

Applications



The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

As each sack is removed, the platform will *rise* slightly since some of the potential energy within the springs will be transformed into an increase in gravitational potential energy of the remaining sacks.

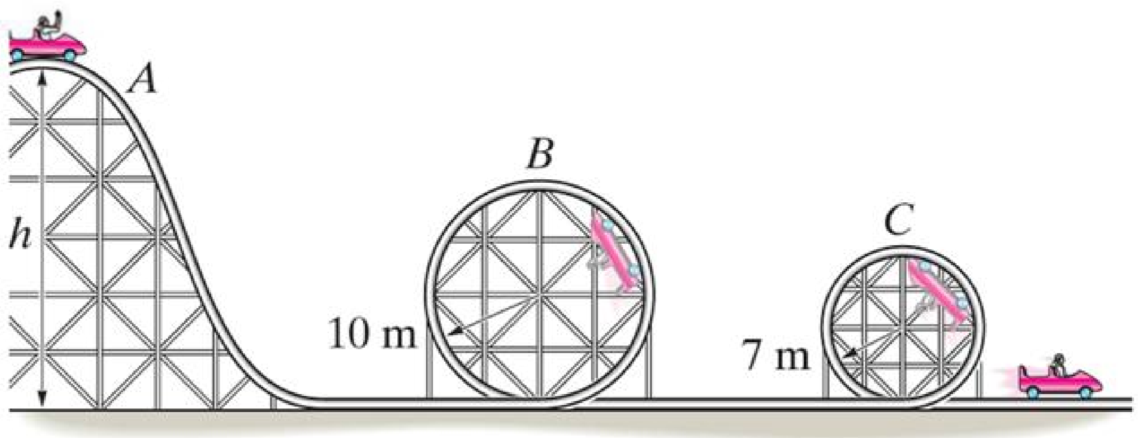
If the sacks weigh 100 lb and the equivalent spring constant is k = 500 lb/ft, what is the energy stored in the springs?

Applications (continued)

The young woman pulls the water balloon launcher back, stretching each of the four elastic cords.

If we know the unstretched length and stiffness of each cord, can we estimate the maximum height and the maximum range of the water balloon when it is released from the current position? Would we need to know any other information?

Applications (continued)



The roller coaster is released from rest at the top of the hill A. As the coaster moves down the hill, potential energy is transformed into kinetic energy.

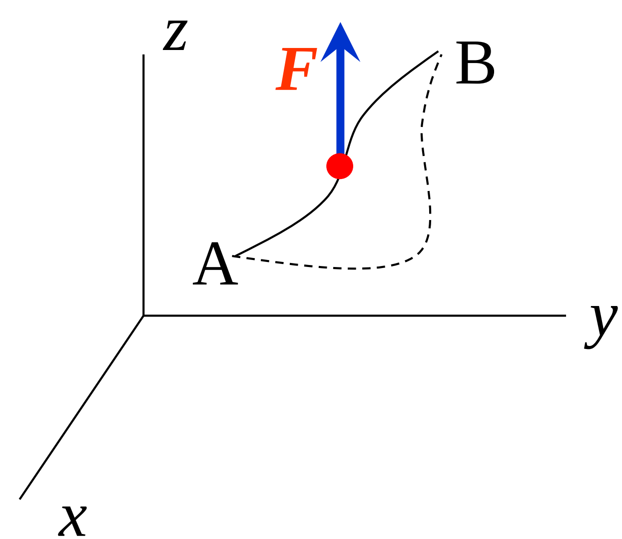
What is the velocity of the coaster when it is at B and C?

Also, how can we determine the minimum height of hill A so that the car travels around both inside loops without leaving the track?

Conservative Force

A force ***F*** is said to be conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B. This also means that the work done by the force ***F*** in a closed path (*i.e.*, from A to B and then back to A) is zero.

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$



Thus, we say the work is conserved.

The work done by a conservative force depends only on the positions of the particle, and is independent of its velocity or acceleration.

Conservative Force (continued)

A more rigorous definition of a conservative force makes use of a potential function (V) and partial differential calculus, as explained in the text. However, even without the use of the these more complex mathematical relationships, much can be understood and accomplished.

The "conservative" potential energy of a particle/system is typically written using the potential function V. There are two major components to V commonly encountered in mechanical systems, the potential energy from gravity and the potential energy from springs or other elastic elements.

$$V_{total} = V_{gravity} + V_{springs}$$

Potential Energy

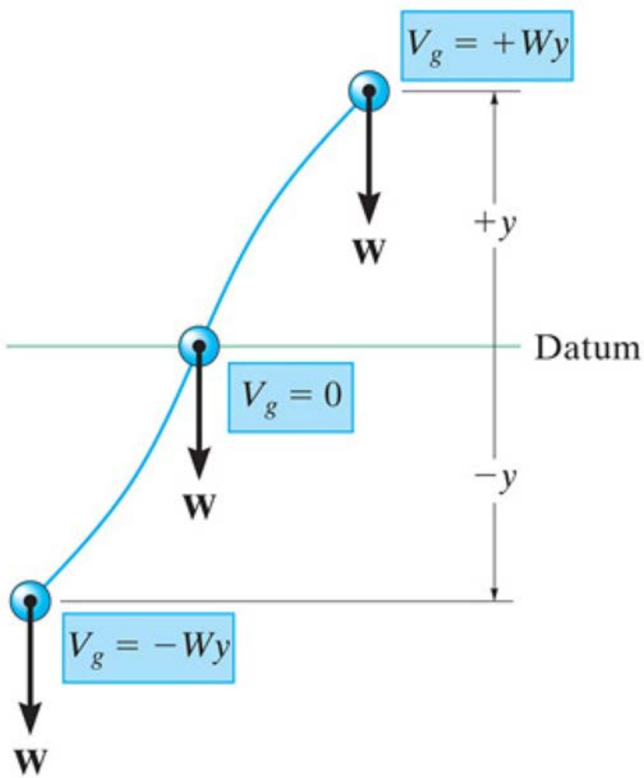
Potential energy is a measure of the amount of work a conservative force will do when a body changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (e.g., the sum of $V_{gravity}$ and $V_{springs}$).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

Potential Energy Due to Gravity

The potential function (formula) for a gravitational force, e.g., weight (W = mg), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

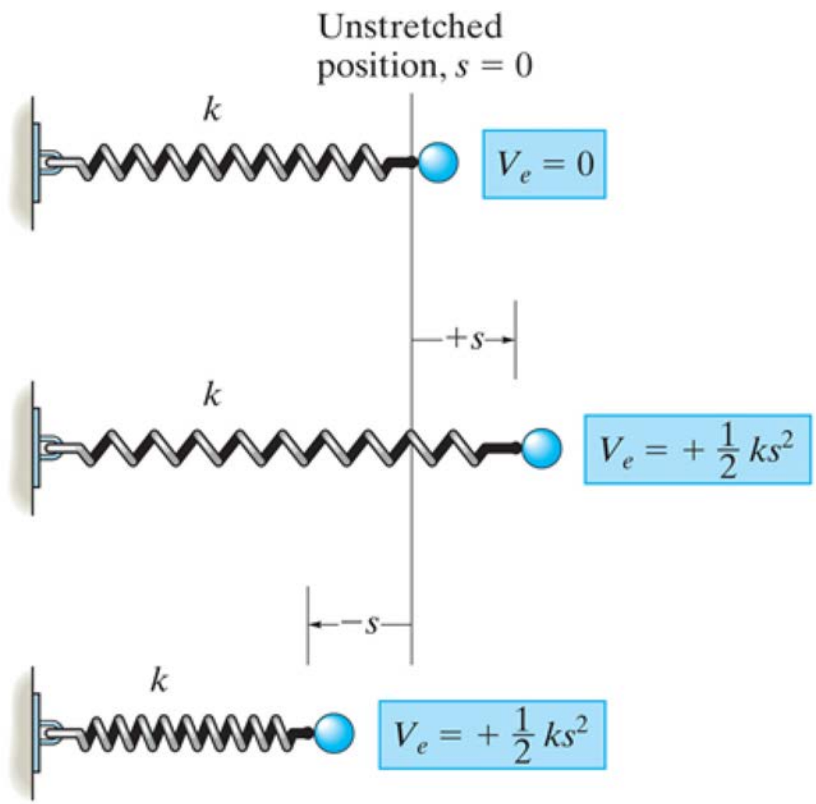


$$V_g = \pm W y$$

V_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum.

Elastic Potential Energy

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.



V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2} k\, s^2$$

Notice that the potential function V_e always yields positive energy.

Conservation of Energy

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 \;=\; \text{Constant}$$

T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_2 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = \frac{1}{2} mv^2$.

Examples & Questions

UNIVERSITY of HAWAIʻI® MĀNOA

Learning Catalytics™

- Please sign in:
  - www.learningcatalytics.com