CEE 271 APPLIED MECHANICS II

Lecture 12: Work and Energy

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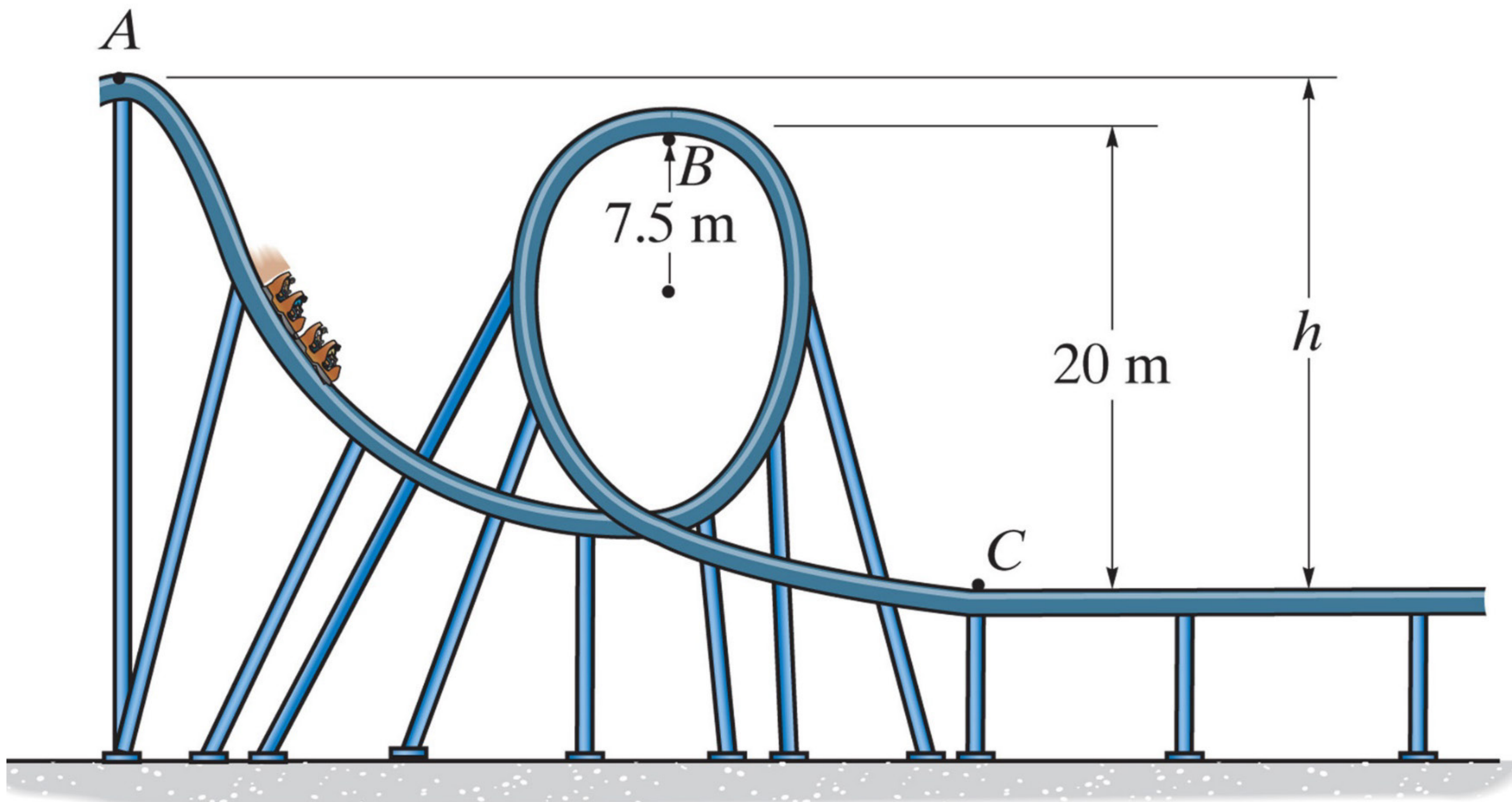
Today's Objectives

- Calculate the work of a force.
- Apply the principle of work and energy to a particle or system of particles.

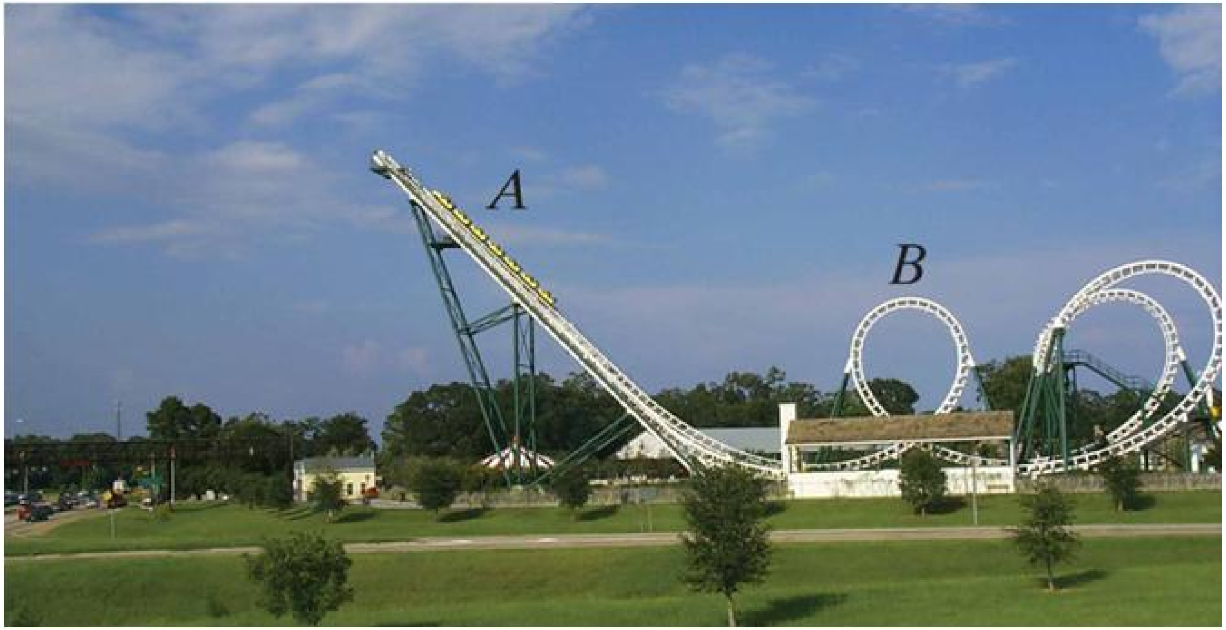
Outline (Pre-Job Brief)

- Work of a Force
- Principle of Work and Energy
- Examples and Questions
- Summary and Feedback

Work and Energy



Applications



A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the "valleys" of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, ρ) to control the forces experienced by the passengers?

Applications (continued)

Crash barrels are often used along roadways in front of barriers for crash protection.

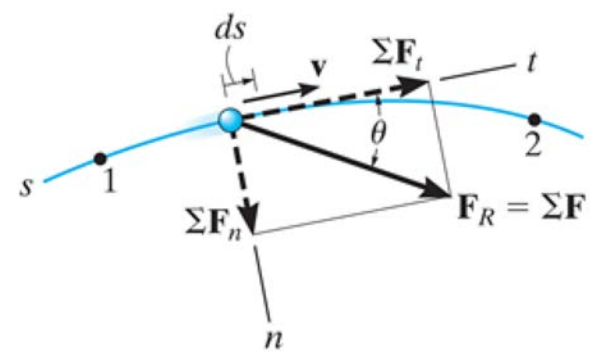
The barrels absorb the car's kinetic energy by deforming.

If we know the velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

Work and Energy

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion ($\boldsymbol{F} = m\mathbf{a}$) with respect to displacement.

By substituting $a_t = v\,(dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy.

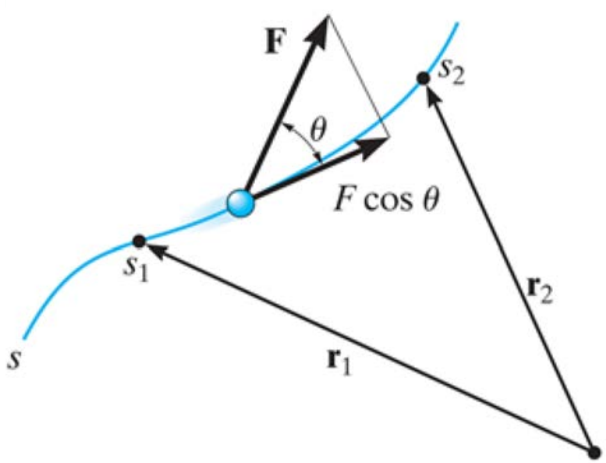


This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power.

To use this principle, we must first understand how to calculate the work of a force.

Work of a Force

A force does work on a particle when the particle undergoes a displacement along the line of action of the force.



Work is defined as the product of force and displacement components acting in the same direction. So, if the angle between the force and displacement vector is θ, the increment of work dU done by the force is

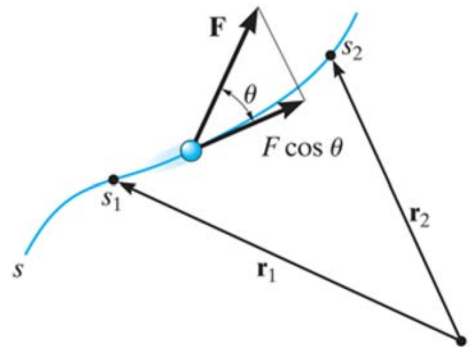
$$dU = F\, ds \cos \theta$$

By using the definition of the dot product and integrating, the total work can be written as

$$U_{1\text{-}2} = \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{F} \bullet d\boldsymbol{r}$$

Work of a Force (continued)

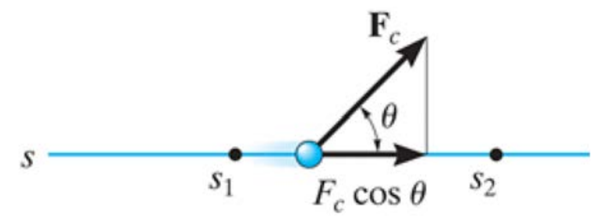
If ***F*** is a function of position (a common case) this becomes



$$U_{1\text{-}2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

If both F and θ are constant ($F = F_c$), this equation further simplifies to

$$U_{1\text{-}2} = F_c \cos\theta \,(s_2 - s_1)$$



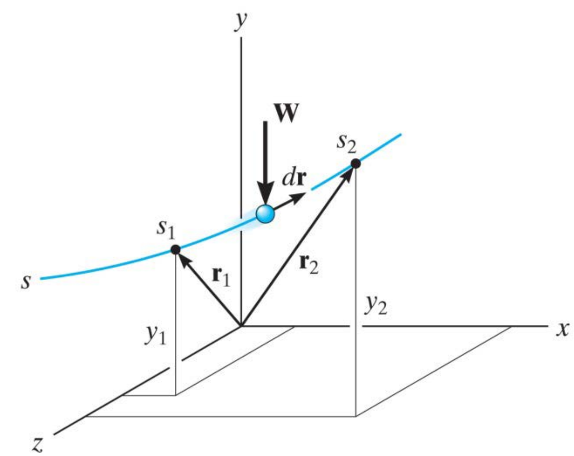
Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero.

Work of a Weight

The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

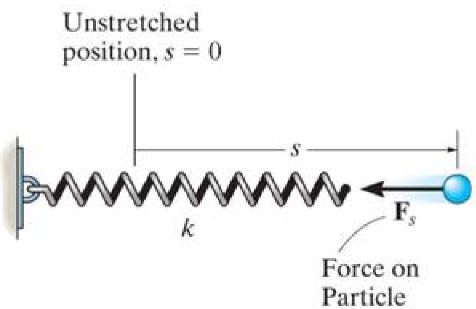
$$U_{1-2} = \int_{y_1}^{y_2} -W\,dy$$

$$U_{1-2} = -W\,(y_2 - y_1) = -W\,\Delta y$$



The work of a weight is the product of the magnitude of the particle's weight and its vertical displacement. If Δy is upward, the work is negative since the weight force always acts downward.

Work of a Spring Force



When stretched, a linear elastic spring develops a force of magnitude $F_s = ks$, where k is the spring stiffness and s is the displacement from the unstretched position.

The work of the spring force moving from position s_1 to position s_2 is

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k\, s\, ds = 0.5\, k\, (s_2)^2 - 0.5\, k\, (s_1)^2$$

If a particle is attached to the spring, the force F_s exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative or

$$U_{1\text{-}2} = -[\, 0.5\, k\, (s_2)^2 - 0.5\, k\, (s_1)^2 \,] \;.$$

Spring Forces

It is important to note the following about spring forces.

1. The equations above are for linear springs only! Recall that a linear spring develops a force according to $F = ks$ (essentially the equation of a line).

2. The work of a spring is not just spring force times distance at some point, i.e., $(ks_i)(s_i)$. Beware, this is a trap that students often fall into!

3. Always double check the sign of the spring work after calculating it. It is positive work if the force on the object by the spring and the movement are in the same direction.

Principle of Work and Energy

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By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, we can derive the principle of work and energy as follows:

$$\sum F_t = ma_t = mv(dv/ds)$$

Integrating both sides:

$$\sum \int_{s_1}^{s_2} F_t ds = \int_{v_1}^{v_2} mv\, dv$$

$$\sum \int_{s_1}^{s_2} F \cos\theta\, ds = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2$$

$$\sum U_{1-2} = \tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 = T_2 - T_1$$

$$T_1 + \sum U_{1\text{-}2} = T_2$$

Principle of Work and Energy

By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, the principle of work and energy can be written as

$$\sum U_{1\text{-}2} = 0.5\, m\, (v_2)^2 - 0.5\, m\, (v_1)^2 \quad \text{or} \quad T_1 + \sum U_{1\text{-}2} = T_2$$

$\sum U_{1\text{-}2}$ is the work done by all the forces acting on the particle as it moves from point 1 to point 2. Work can be either a positive or negative scalar.

T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively. Thus, $T_1 = 0.5\, m\, (v_1)^2$ and $T_2 = 0.5\, m\, (v_2)^2$. The kinetic energy is always a positive scalar (velocity is squared!).

So, the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy.

Work and Energy

Note that the principle of work and energy ($T_1 + \sum U_{1\text{-}2} = T_2$) is not a vector equation! Each term results in a scalar value.

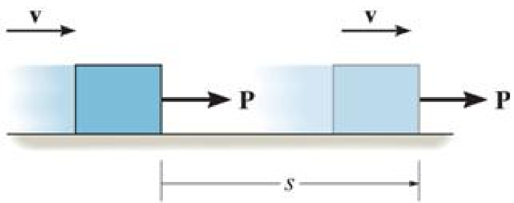
Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb.

The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work.

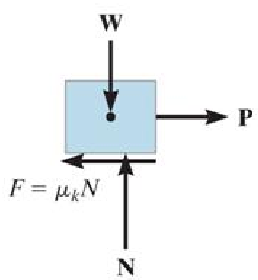
The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

Work of Friction: Sliding

The case of a body sliding over a rough surface merits special consideration.



Consider a block which is moving over a rough surface. If the applied force ***P*** just balances the resultant frictional force $\mu_k N$, a constant velocity v would be maintained.



The principle of work and energy would be applied as

$$0.5m\,(v)^2 + P\,s - (\mu_k\,N)\,s = 0.5m\,(v)^2$$

This equation is satisfied if $P = \mu_k N$. However, we know from experience that friction generates heat, a form of energy that does not seem to be accounted for in this equation. It can be shown that the work term $(\mu_k N)s$ represents both the external work of the friction force and the internal work that is converted into heat.

Examples & Questions

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